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Finite Sample Properties of Impulse Response Intervals in SVECMs with Long-Run Identifying Restrictions

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Finite Sample Properties of Impulse Response Intervals in SVECMs with Long-Run Identifying Restrictions*

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Abstract

This paper investigates the finite sample properties of confidence intervals for structural vector error correction models (SVECMs) with long-run identifying restrictions on the impulse response functions. The simulation study compares methods that are frequently used in applied SVECM studies including an interval based on the asymptotic distribution of impulse responses, a standard percentile (Efron) bootstrap interval, Hall's percentile and Hall's studentized bootstrap interval. Data generating processes are based on empirical SVECM studies and evaluation criteria include the empirical coverage, the average length and the sign implied by the interval. Our Monte Carlo evidence suggests that applied researchers have little to choose between the asymptotic and the Hall bootstrap intervals in SVECMs. In contrast, the Efron bootstrap interval may be less suitable for applied work as it is less informative about the sign of the underlying impulse response function and the computationally demanding studentized Hall interval is often outperformed by the other methods. Differences between methods are illustrated empirically by using a data set from King, Plosser, Stock & Watson (1991).

Keywords: Structural vector error correction model, impulse response intervals, cointegration, long-run restrictions, bootstrap.

JEL classification: C32, C53, C15

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1 Introduction

The analysis of impulse responses in the framework of vector autoregressive (VAR) models has become one of the dominant tools in empirical macroeconomic analysis. Interest usually focuses on the dynamic effects of different macroeconomic shocks (e.g. monetary policy shocks, fiscal shocks, oil price shocks). In the literature on structural VAR (SVAR) models, these shocks are typically recovered from reduced form VAR models by imposing identifying restrictions motivated by economic theory on the structure of the covariance matrix. Structural impulse response functions (IRFs) are computed to investigate the effects of the economically meaningful shocks. In contrast to the first generation SVAR models that have been primarily based on unrestricted VAR models, recent structural analysis is often based on cointegrated VAR and vector error correction models (VECMs). Following the work by King et al. (1991) a number of studies have modeled the reduced form as a vector error correction mechanism. Using VECMs instead of unrestricted VARs has been also advocated by Phillips (1998) because the latter models produce inconsistent estimates of impulse responses and forecast error variances at long horizons. The resulting models are called structural vector error correction models (SVECMs) as the structural analysis is based on a VECM. A key feature of the SVECM class is that some structural shocks are identified by long-run restrictions which are directly related to the cointegration properties of the data (see e.g. Breitung, Brüggemann & Lütkepohl (2004) for a recent overview of SVECMs). Consequently, the SVECM class is closely related to the literature on permanent-transitory decompositions (see e.g. Levtchenkova, Pagan & Robertson (1998) for an excellent overview and Gonzalo & Ng (2001)). Empirical SVECM studies include *inter alia* Mellander, Vredin & Warne (1992), Fisher, Fackler & Orden (1995), Coenen & Vega (1999), Fisher, Huh & Tallman (2003), Ribba (2003a, 2003b), Jang & Ogaki (2004) and Vlaar (2004b).

For the interpretation of structural IRFs it has also become standard practice to report confidence intervals (CIs) around the point estimates to assess the estimation uncertainty. Different methods for the construction of IRF intervals have been suggested in the literature. CIs may be based on the asymptotic distributions of the impulse responses (see Lütkepohl (2005)), on Monte Carlo integration methods of Sims & Zha (1999) and on various variants of bootstrap methods (see e.g. Kilian (1998c) and Benkwitz, Lütkepohl & Wolters (2001)). In the context of SVECMs with long-run restrictions four methods have been primarily used in applied work: The first one is based on a generalization of the asymptotic intervals given in Lütkepohl & Reimers (1992) suggested by Vlaar (2004a). In the presence of long-run restrictions a correction of the asymptotic distribution is needed which takes the stochastic nature of the identifying restrictions into account. Empirical applications of this method include Coenen & Vega (1999) and Vlaar (2004b). As an alternative to the asymptotic intervals, bootstrap methods have been used in the context of SVECM. In particular, the standard percentile interval of Efron & Tibshirani (1993), the Hall percentile interval and the studentized Hall interval (see Hall (1992)) have been used in Lütkepohl & Wolters (2003), Brüggemann (2004) and Breitung et al. (2004). These three bootstrap versions are available for SVECMs with long-run restrictions in form of the menu driven software JMulTi (see www.jmulti.com) and may be readily applied by interested researchers.

To date very little is known on the finite sample properties of these CI construction methods in the context of SVECMs with long-run restrictions. Available Monte Carlo evidence as e.g. in Kilian (1998a, 1998b, 1998c, 2001) and Kilian & Chang (2000) is essentially limited to unrestricted VAR models with contemporaneous identifying restrictions based on a Choleski decomposition of the covariance matrix. Possibly because they require numerical optimization that are computationally burdensome, SVAR and especially SVEC models with more general identification schemes (including e.g. long-run restrictions) have not yet been considered in simulation studies.

In this paper we fill the gap and compare the finite sample properties of the described CI construction methods for SVECMs with long-run restrictions. For this purpose we conduct a Monte Carlo study using a large number of data generating processes (DGPs) obtained by estimating SVECMs from the literature. Our comparison is not only based on standard criteria such as empirical coverage and average CI length but also includes a new criterion that evaluates the ability of a specific CI to indicate the underlying sign of the impulse response function. We argue that indicating the right sign with high probability is an important property of intervals because it is likely to affect the interpretation of structural IRFs more than small differences in coverage rates. Moreover, we distinguish between results for responses to permanent and transitory shocks. This is instructive because permanent and transitory shocks have different implications on the shape of the response functions and, consequently, may affect the properties of the corresponding CIs.

Based on our Monte Carlo evidence, it appears that applied researchers have little to choose between the asymptotic and the Hall bootstrap percentile intervals in SVECMs with long-run restrictions. In contrast, the standard (Efron) percentile bootstrap interval may be less suitable for applied work as it is less informative about the sign of the underlying impulse response function. Comparing CIs for responses to permanent shocks, we find that asymptotic and Hall bootstrap intervals have similar coverage rates and indicate the right sign of the underlying response equally often. Consequently, they allow a similar interpretation. However, the bootstrap CIs are usually asymmetric and much wider. In contrast, the standard (Efron) bootstrap interval includes the zero line more often and therefore indicates the correct sign less often. For responses to transitory shocks the CIs are much more alike. However, we find that the computationally demanding studentized Hall interval is often outperformed by the other methods.

The remainder of the paper is structured as follows. Section 2 introduces the modeling framework and reviews different methods of constructing CIs for the structural IRFs. In Section 3 we present the Monte Carlo design, discuss evaluation criteria and sum up the results of our Monte Carlo comparison. Section 4 illustrates the use of different methods using a small

U.S. macroeconomic model originally analyzed by King et al. (1991) and concluding remarks are given in Section 5.

2 SVEC Models and Impulse Response Confidence Intervals

We analyze confidence intervals for responses to structural shocks within the framework of structural vector error correction models (SVECMs). This modeling approach takes explicitly cointegration restrictions into account and long-run restrictions can be used to identify the structural shocks (see Breitung et al. (2004) for recent overview of this modeling class). In the following, we assume that all variables in the K-dimensional vector y_t are at most integrated of order 1, denoted as $I(1)$, and that the times series can be well described by a VEC model with cointegration rank r given by

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + \alpha\beta' y_{t-1} + \mathsf{B}\varepsilon_t, \tag{2.1}$$

where y_t is a vector of observable variables, α is a $K \times r$ matrix of loading coefficients, β is the $K \times r$ matrix containing the cointegration vectors, and $\Gamma_1, \ldots, \Gamma_{p-1}$ are $K \times K$ coefficient matrices. The reduced form error terms $u_t = \mathsf{B}\varepsilon_t$, i.e. u_t is expressed as linear combinations of the structural shocks ε_t. Moreover, we assume that u_t is a white noise error vector with zero mean and time invariant covariance matrix Σ_u. The structural shocks ε_t are assumed to be mutually uncorrelated, such that $E[\varepsilon_t \varepsilon_t']$ is diagonal. Without loss of generality we normalize the variances of the structural shocks to unity such that $\Sigma_\varepsilon = I_K$. Using Granger's representation theorem (see Johansen (1995, Theorem 4.2)), it can be shown that the long-run effects of u_t are given by the total impact matrix

$$C(1) = \beta_\perp (\alpha_\perp' (I_K - \sum_{i=1}^{p-1} \Gamma_i) \beta_\perp)^{-1} \alpha_\perp',$$

where $\beta_\perp$ and $\alpha_\perp$ represent the orthogonal complements of β and α, respectively. It follows from (2.1) that the long-run effects of structural shocks ε_t can be written as

$$C(1)\mathsf{B}. \tag{2.2}$$

Note that $C(1)$ has reduced rank rk$(C(1)) = K - r$ and consequently there are $K - r$ common trends. This implies that at most r shocks may have transitory effects because the matrix $C(1)\mathsf{B}$ cannot have more than r columns of zeros. Thus, the cointegration rank r of the system is informative with respect to the maximum number of transitory shocks.

Identifying restrictions are needed for estimating the contemporaneous impact matrix B and in the context of our modeling framework it is possible to use restrictions on B (short-run or contemporaneous restrictions) and restrictions on $C(1)\mathsf{B}$ (long-run restrictions). Here we only consider the case of exclusion restrictions that may be written for the short-run restrictions as

$$R_s \text{vec}(\mathsf{B}) = 0$$

and for the long-run restrictions as

$$R_l \text{vec}(C(1)\mathsf{B}) = R_l(I_K \otimes C(1))\text{vec}(\mathsf{B}) = R_l^* \text{vec}(\mathsf{B}) = 0.$$

Note that we have rewritten the long-run restrictions such that they can be expressed as linear restrictions on vec(B) but the restriction matrix R_l^* now involves the elements of $C(1)$. In practice, $C(1)$ has to be replaced by an estimate and, consequently, the restriction matrix R_l^* is stochastic. It is this stochastic nature of the restriction matrix that makes a modification of the asymptotic confidence bands necessary (see Vlaar (2004a) and Section 2.1).

Estimates for the contemporaneous impact matrix can be found by maximizing the concentrated log-likelihood function given by

$$\ln l(\mathsf{B}) = \text{constant} - \frac{T}{2} \ln |\mathsf{B}|^2 - \frac{T}{2} \text{tr}\left((\mathsf{B}')^{-1} \mathsf{B} \widetilde{\Sigma}_u \right), \tag{2.3}$$

with respect to the free structural parameters subject to the identifying restrictions, where $\widetilde{\Sigma}_u$ is the estimated residual covariance matrix from the reduced form VECM (see Breitung et al. (2004) for more details).

The quantities of interest in empirical studies are often given by the impulse response functions derived from the SVEC. To compute the structural impulse response functions from the SVEC, we transform (2.1) into the corresponding levels version by letting $A_1 = \Gamma_1 + \alpha\beta' + I_K$, $A_i = \Gamma_i - \Gamma_{i-1}$ for $i = 2, \ldots, p-1$ and $A_p = -\Gamma_{p-1}$ such that

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t. \tag{2.4}$$

Then the response functions to an impulse in the reduced form (forecast error) u_t are given by letting $\Phi_0 = I_K$ and

$$\Phi_i = \sum_{j=1}^{i} \Phi_{i-j} A_j, \qquad i = 1, 2, \ldots \tag{2.5}$$

Consequently, the structural impulse response functions are given by

$$\Theta_i = \Phi_i \mathsf{B}, \qquad i = 0, 1, 2, \ldots \tag{2.6}$$

In practice, SVEC models are typically estimated by a two-step procedure. In the first step, the reduced form VECM is specified and estimated. Given an estimate for the reduced form covariance matrix Σ_u and enough identifying restrictions, an estimate of B can be obtained by ML estimation. Moreover, estimates of $A_j, j = 1, \ldots, p$ are easily obtained from the reduced form VEC parameter estimates. Consequently, the estimated impulse responses are a function of estimated parameters, hence, also estimates. To assess the uncertainty around the impulse response estimates, confidence bands are typically plotted around the point estimates. Four methods that are often used in practical SVECM studies are reviewed in the following sections.

2.1 Asymptotic Impulse Response Intervals

The estimation uncertainty around the estimated impulse response may be quantified by using asymptotic distribution results. If the reduced form parameters are asymptotically normally distributed, the estimated structural impulse responses $\hat{\Theta}_i$ will also have an asymptotic normal distribution as $\hat{\Theta}_i$ is a nonlinear function of the reduced form parameters. Consequently, the asymptotic distribution of the structural impulse response may be obtained by using the delta method. For instance, the asymptotic distribution of impulse responses for SVAR models that are not based on cointegrated VAR and VEC models has been derived by Amisano & Giannini (1997). For the case of forecast error and orthogonalized impulse responses in cointegrated VAR model, the asymptotic distribution is given in Lütkepohl & Reimers (1992). In comparison with results given in those studies, the derivation of the asymptotic distribution has to be modified if long-run restrictions on the effects of structural shocks are considered. The modification is needed because the long-run restrictions imply a stochastic restriction matrix. Vlaar (2004a) derives the asymptotic distribution that is relevant for the considered SVEC models. Vlaar (2004a) shows that

$$\sqrt{T}\text{vec}(\hat{\Theta}_i - \Theta_i) \rightarrow N(0, \Sigma_{\Theta_i}), \tag{2.7}$$

where

$$\Sigma_{\Theta_i} = (F_i + G_i)\Sigma_\gamma(F_i + G_i)' + H_i\Sigma_{\gamma_\mathsf{B}}H_i', \tag{2.8}$$

Σ_γ is the covariance matrix of the VEC parameters $\gamma = \text{vec}(\Gamma_1, \Gamma_2, \ldots, \Gamma_{p-1}, \alpha)$. Moreover, $\Sigma_{\gamma_\mathsf{B}}$ is the covariance matrix of the free structural parameters γ_B. Detailed expressions for F_i, H_i and G_i are given in the Appendix A. If the unknown quantities in (2.8) are replaced by estimates, asymptotic confidence bands for the elements of $\hat{\Theta}_i$ may be based on the estimated standard deviations that are given by the square roots of the diagonal elements of $T^{-1}\hat{\Sigma}_{\Theta_i}$. For instance, an asymptotic 95% interval is given by adding ± 1.96 estimated standard deviations to the point estimate. Consequently, the asymptotic CIs are necessarily symmetric around the point estimate. The asymptotic CI will be denoted as CI_A in the following.

Vlaar's modification essentially introduces the additional matrix G_i in (2.8). Using an empirical example, Vlaar (2004a) illustrates that for transitory shocks neglecting the stochastic nature of the restrictions (i.e. setting $G_i = 0$ in (2.8)) '*one mistakenly assumes very precise predictions at short horizons, whereas at long horizons the confidence bands do not converge to zero*'.

2.2 Bootstrap Intervals

In recent years, inference on impulse response functions that are based on bootstrap methods have become increasingly popular because they sometimes have better small sample properties than intervals based on asymptotic theory (see e.g. Kilian (1998c)). Moreover, they are easy to compute given presently available computing technology and avoid the relatively complicated analytical expressions for the asymptotic covariance matrix (see Section 2.1).

Alternative residual-based bootstrap methods have been suggested in the literature (see Benkwitz et al. (2001) for an overview). In the context of VEC models, the bootstrap procedures work as follows: The model (2.1) is estimated and the estimation residuals $\hat{u}_t$ are centered around their mean $\bar{\hat{u}}$. Bootstrap residuals $u_1^*, \dots, u_T^*$ are generated by drawing randomly with replacement from the centered residuals, $\hat{u}_1 - \bar{\hat{u}}, \dots, \hat{u}_T - \bar{\hat{u}}$. The bootstrap residuals together with the estimated model parameters and the given presample values are used to generate bootstrap time series. The model is reestimated using the bootstrap time series and the quantities of interest, in our case the structural impulse responses, are determined on the basis of the bootstrap estimates.[1] Repeating these steps many times gives the empirical bootstrap distribution of the impulse response functions. Using this distribution, CIs may be obtained for $\hat{\Theta}_i$.

In this paper we consider three types of bootstrap confidence intervals that have been suggested in the literature (see Benkwitz, Lütkepohl & Neumann (2000) and Benkwitz et al. (2001)), are readily available for applied researchers in the form of a menu driven software[2] and have often been used in applied SVECM studies with long-run identifying restrictions (see *inter alia* Brüggemann (2004) and Breitung et al. (2004)). We denote by θ, $\hat{\theta}$, and $\hat{\theta}^*$ a general impulse response coefficient, its estimator based on the estimated model coefficients and the corresponding bootstrap estimator, respectively. The three bootstrap methods are given in the following:

Standard percentile interval. The first bootstrap method is based on the standard percentile interval (see Efron & Tibshirani (1993)):

$$CI_S = \left[s^*_{\xi/2}, s^*_{(1-\xi/2)}\right], \tag{2.9}$$

where $s^*_{\xi/2}$ and $s^*_{(1-\xi/2)}$ are the $\xi/2$ and $(1-\xi/2)$-quantiles of the empirical bootstrap distribution of $\hat{\theta}^*$. We will refer to this interval as the Efron interval in the following.

Hall's percentile interval. The second method is based on the interval presented in Hall (1992) which is given by

$$CI_H = \left[\hat{\theta} - t^*_{(1-\xi/2)}, \hat{\theta} - t^*_{\xi/2}\right], \tag{2.10}$$

where $t^*_{\xi/2}$ and $t^*_{(1-\xi/2)}$ are the quantiles of the empirical distribution of $(\hat{\theta}^* - \hat{\theta})$. We will refer to this interval as the Hall interval in the following.

[1] In our implementation of the bootstrap the estimated cointegration matrix β is fixed in all bootstrap replications and only the remaining parameters of the VECM are reestimated in every bootstrap replication. Benkwitz et al. (2001) find that reestimating β in every bootstrap replication leads to similar results.

[2] Both considered bootstrap methods are implemented in the software JMulTi (see www.jmulti.com and Lütkepohl & Krätzig (2004)).

Hall's studentized interval. The third method uses a studentized statistic presented in Hall (1992). The confidence interval is constructed by using the bootstrap quantiles $t^{**}_{\xi/2}$ and $t^{**}_{(1-\xi/2)}$ from the distribution $(\hat{\theta}^* - \hat{\theta})/(\widehat{\text{var}}(\hat{\theta}^*))^{1/2}$ to compute the interval

$$CI_{SH} = \left[\hat{\theta} - t^{**}_{(1-\xi/2)}\sqrt{\widehat{\text{var}}(\hat{\theta})},\ \hat{\theta} - t^{**}_{\xi/2}\sqrt{\widehat{\text{var}}(\hat{\theta})}\right], \tag{2.11}$$

where the variances may be estimated by a bootstrap within each bootstrap replication (see e.g. Lütkepohl (2005, Appendix D.3)). We will refer to this interval as the studentized Hall interval in the following. Due to the inner bootstrap in each bootstrap replication, this methods is computationally very demanding. We have included this interval in our study as this method should result in more precise confidence intervals (at least in theory).

Note that to date there is no formal proof for the described bootstrap procedures to be consistent in the context of cointegrated VAR, VEC and SVEC models. Including them in our study is therefore mostly motivated by the fact that bootstrap methods have been frequently used by applied researchers. Also note that the bootstrap methods 'automatically' take the long-run restrictions into account, i.e. a modification for the case of long-run restrictions (as for the asymptotic intervals) is not needed here.

Another potential problem of the bootstrap in the context of SVEC models is related to the fact that we have to use numerical optimization for estimating the structural parameters of the contemporaneous impact matrix B in every bootstrap replication. Consequently, there may be some bootstrap replications in which the algorithm does not converge. In the past, this limited the application of bootstrap methods to models that could be estimated without numerical optimization, e.g. when the Choleski decomposition of Σ_u is used. We have solved this computational issue by using a fast and robust estimation algorithm. Our algorithm uses the original point estimate as a starting value in every bootstrap replication and usually results in convergence after a few iterations.[3] Consequently, it is now easily possible to apply the bootstrap methods to set up CIs for the structural impulse responses of a SVECM.

3 Monte Carlo Comparison

3.1 Monte Carlo Design

The simulation study uses DGPs that are obtained from estimating SVECMs previously specified in the empirical literature. Using empirical model specifications leads to DGP characteristics, such as number of endogenous variables, lag length and number of cointegration properties,

[3] In the rare event of no convergence after 500 iterations, we try up to 50 different random starting values. If there is still no convergence, the corresponding bootstrap draw will be deleted. This typically happens only once or twice in 2000 bootstrap replications. This extremely fast and robust version of the SVEC algorithm has been implemented in Gauss and is also used by the software package JMulTi.

similar to those typically encountered by applied time series econometricians. We think that the results of this strategy are more useful to practitioners than presenting results on relatively simple, artificial DGPs. Indeed, we have used a large number of DGP specifications that are based on some of the studies cited in the introduction. Our Monte Carlo study is based on the DGPs whose properties are summarized in Table 1, where we give the cointegration structure and the identification scheme together with the moduli of the VAR companion matrix.

DGP (A) is obtained by using the estimates of a model for the log of consumption c_t, the log of investment i_t and the log of private output q_t as specified by King et al. (1991). Following their analysis, the data are modeled by imposing a cointegration rank of two ($r = 2$) in the three-dimensional system. The DGP parameters are obtained by imposing the balanced growth path conditions (see β in Table 1) before estimating a VECM with eight lagged differences and an unrestricted constant (as in the original study). DGP (B) is a variant of DGP (A) that is obtained by using only $p = 2$ lags, which is sufficient according to standard information criteria. Structural identification of the permanent shock follows the original study. In addition, we disentangle the two transitory shocks by a zero restriction in B.

DGP (C) corresponds to the six-variable model in King et al. (1991) which in addition to c_t, i_t and q_t also includes the log of real money m_t, an interest rate R_t and the inflation rate Δp_t. The cointegration structure ($r = 3$) and the long-run restrictions on $C(1)\mathsf{B}$ correspond to the original study. The remaining parameters have been obtained by estimating a VECM with eight lagged differences and an unrestricted constant (as in the original study). Moreover, we have imposed a recursive structure on B to identify the three transitory shocks. DGP (D) is a variant of DGP (C) with a smaller number of lags ($p = 2$). The length of the generated time series in the Monte Carlo study in DGPs (A) to (D) is $T = 168$ and corresponds to the number of observations used in the original study.

DGP (E) is a five-variable structural VECM obtained from a small monetary system of the Euro area provided by Coenen & Vega (2001). The long-run identifying assumptions are driven by the cointegration properties ($r = 3$), while the contemporaneous restrictions are merely imposed for convenience. The remaining parameters have been obtained by estimating a VECM with one lag of Δy_t and an unrestricted constant. Generated time series of length $T = 75$ have been used in the simulation.

DGP (F) is a SVECM for inflation π_t and unemployment U_t based on Ribba (2003b). The parameters are obtained by imposing $\beta = (1, -1)$ and then estimating a VECM with six lagged differences ($p = 7$) and an unrestricted constant. Generated time series of length $T = 371$ have been used in the simulation. A zero column on the long-run impact matrix separates the transitory and the permanent shock in the system. No further restrictions are needed in this bivariate system because there is only one permanent and one transitory shock. Note that DGPs (A) to (E) are based on models for quarterly data, while DGPs (F) and (G) have been obtained from monthly data.

For each design point in our Monte Carlo experiment we have generated $M = 1000$ sets of time series by drawing normally distributed errors from multivariate distributions with zero mean and covariance matrices corresponding to the VECMs in Table 1. We have used zero initial values and have truncated the first 50 observations to eliminate the impact of the starting values. The length of the generated time series corresponds to that of the time series used in the original studies. For each Monte Carlo replication we computed the structural impulse response functions together with the three bootstrap and the asymptotic confidence intervals. The bootstrap intervals are based on 500 bootstrap replications and 100 inner bootstrap replications for the studentized Hall interval in order to obtain reasonable computation times.[4] All computations are done with GAUSS.

3.2 Evaluation Criteria

We use three evaluation criteria to assess the accuracy of different impulse response CIs. The first criterion is the empirical coverage of impulse response confidence intervals as suggested by *inter alia* Kilian (1998a), Kilian & Chang (2000), and Kilian (2001). More formally, let $\hat{\theta}_{jk,h}$ be the estimated structural response of variable j to an impulse in variable k, h periods ago. Then denote a $(1-\alpha)\times 100\%$ confidence interval around $\hat{\theta}_{jk,h}$ by $[\gamma_{\frac{\alpha}{2},jk,h};\gamma_{1-\frac{\alpha}{2},jk,h}]$. The empirical coverage of that interval can be computed from a Monte Carlo experiment and is given by

$$\mathsf{EC}_{jk,h} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{I}(\gamma_{\frac{\alpha}{2},jk,h,m} \leq \theta_{jk,h} \leq \gamma_{1-\frac{\alpha}{2},jk,h,m}), \tag{3.1}$$

where $\mathbf{I}(\cdot)$ is an indicator function that takes the value one, if the true DGP impulse response $\theta_{jk,h}$ is within the interval of replication m. M indicates the number of Monte Carlo replications. Ideally, the empirical coverage should be close to the nominal level of $1-\alpha$.

Secondly, we look at the average length (AL) of the confidence intervals which can be computed as

$$\mathsf{AL}_{jk,h} = \frac{1}{M}\sum_{m=1}^{M}(\gamma_{1-\frac{\alpha}{2},jk,h,m} - \gamma_{\frac{\alpha}{2},jk,h,m}). \tag{3.2}$$

Clearly, one would expect that the interval length is related to its coverage properties. But judging empirical coverage and average length together may reveal information on differences of the interval locations. In our context this is a useful information because the $\pm$ standard error asymptotic confidence bands are necessarily symmetric while the shape of the bootstrap intervals depends on the bootstrap distribution and may as well be asymmetric.

Our third evaluation criterion is related to the sign implied by the respective confidence interval. In interpreting impulse response functions, the sign of the response is one of the most important quantities of interest. Typically, the description includes statements about the sign and

[4]We have repeated some of our simulations with 1000 bootstrap draws. The results are virtually identical to the ones discussed here.

the significance of responses to different shocks. To assess how different CI methods capture the sign of the underlying impulse response, we need to compare the sign implied by a particular method to a measure of the 'true' sign as implied by the DGP. Clearly, different measures of the 'true sign' are possible. Instead of using the sign given by the DGP impulse response function, we use a more fuzzy measure. In particular, we use the sign which is implied by a 90% confidence interval around the DGP impulse response. Let $S(\theta_{jk,h})$ denote the sign implied by this interval. Then the 'true' impulse response is said to have a zero impact when the zero line is included in this interval, denoted as $S(\theta_{jk,h}) = 0$. Accordingly, the 'true' sign of the impulse response $\theta_{jk,h}$ is positive (negative) when the both interval limits are above (below) the zero line. These situations are denoted as $S(\theta_{jk,h}) = 1$ and $S(\theta_{jk,h}) = -1$, respectively. Accordingly, the implied sign of an interval for $\hat{\theta}_{jk,h}$ computed by a particular method in replication m is denoted as $S(\hat{\theta}_{jk,h})_m$. For each CI construction method we record the relative frequency of same implied signs as

$$\mathsf{CS}_{jk,h} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{I} \left(S(\hat{\theta}_{jk,h})_m = S(\theta_{jk,h}) \right) \tag{3.3}$$

where $\mathbf{I}(\cdot)$ is an indicator function that takes the value one, if the sign implied in replication m equals the sign implied by the DGP. Values of $\mathsf{CS}_{jk,h}$ close to 1 point to CI methods that capture the true dynamics quite well.

3.3 Monte Carlo Results

The following Monte Carlo results have been obtained by estimating correctly specified SVEC models. In other words, results are based on using the correct lag length p, the correct cointegration rank r and deterministic terms as specified in Table 1. Structural parameters are estimated by using the corresponding identification scheme from Table 1. Our simulation results are summarized in Figures 1 and 2 as well as in Tables 2 and 3.

We start the discussion by presenting some typical results in graphical form. Figure 1 shows the empirical coverage of the considered methods obtained from simulations of DGP (A) using a nominal coverage of 90% which is indicated by the solid line. The graph shows in the jk-th panel, $j, k = 1, \ldots, K$, the results for the response of variable j to an impulse in variable k (a structural shock associated with variable k). Our identification scheme for DGP (A) leads to one permanent and two transitory shocks and the results for the permanent shock are given in the left column of the figure which we discuss first.

All four methods lead to intervals that have empirical coverage below nominal. The bootstrap variants are typically slightly closer to the nominal level, especially for larger impulse response horizons. Moreover, CI_{SH} has slightly higher coverage than CI_H and in turn, CI_H has slightly higher coverage than the Efron bootstrap interval. It is instructive to check the average length of the corresponding intervals that are given in the first column of Figure 2. By construction the lengths of two bootstrap methods CI_S and CI_H are equal. Note, that the higher coverage values of the bootstrap intervals coincide with much broader confidence bands for the permanent shocks such that some part of the difference is explained by the length of the interval and not by its location. This is particularly evident for the studentized Hall interval CI_{SH}. Similar coverage and considerably wider bootstrap intervals imply that there must be substantial overlapping of both interval types.

The situation is somewhat different for intervals of responses to transitory shocks (see e.g. the second and third column in Figures 1 and 2). In DGP (A) empirical coverage is higher than for responses to permanent shocks (exceptions are a few responses on impact), often even higher than the nominal coverage for large forecasting horizons. This may be explained by the fact that the long-run responses are restricted to zero. In this case, also the intervals are restricted and therefore they include the true impulse response function with higher probability. Moreover, it now seems that CI_S is slightly better than the asymptotic interval CI_A which in turn is slightly better than CI_H. Note that the studentized Hall interval has typically the lowest coverage rates in this situation. Interestingly, it is now the asymptotic interval that is somewhat longer than the bootstrap intervals CI_S and CI_H, however, the average length of both methods are much closer together than in the case of permanent shocks. Also note that the asymptotic and the studentized Hall interval have about the same length in this example.

As we have a relatively large number of DGPs, we choose to summarize the main results in a few tables instead of reporting graphs for all the results obtained. The graphical representation of results for DGP (A) in the foregoing suggests that the relative accuracy of the considered methods depends on the forecasting horizon h and on the shape of the impulse response function (transitory vs. permanent shocks). Therefore, we structure the analysis of our results accordingly. To begin with, we report average results for all DGPs and different forecasting horizons h. We list in Table 2 the average coverage rates, the average lengths and the relative frequencies of correct implied signs for nominal 90% asymptotic and bootstrap confidence intervals obtained by averaging the quantities over all response functions but over different forecasting horizons h. For instance, the first panel shows for each DGP averages of the results for all $\hat{\theta}_{jk,h}$ over all horizons $h = 0, \ldots 48$ and all impulse responses $j, k = 1, \ldots, K$, while in the second panel shows results for $h = 0, \ldots, 12$.

The overall average coverage values in the first panel suggest that the four considered methods usually lead to intervals with empirical coverage lower than the nominal level. In some cases, especially for DGPs with many variables (e.g. DGP (D) and (E)), the empirical coverage is substantially lower (51% compared to 90% nominal coverage), implying that the underlying estimation uncertainty will be understated. There is not much to choose between the asymptotic CI_A and the Hall interval CI_H. Average coverage for both methods are typically very similar, an exception is given for DGP (B) where the asymptotic intervals is notably closer to the nominal level. Interestingly, the studentized Hall interval, CI_{SH}, has typically lower coverage rates than the Hall percentile interval (CI_H). In contrast, the Efron interval CI_S has the highest average empirical coverage in all considered cases. With exception to the two-variable

DGP (F), the bootstrap intervals are on average somewhat wider. The coverage advantage of CI_S in comparison to CI_H can only be explained by its different location, while location and length may be the source of the relative advantage with respect to the asymptotic interval CI_A and the studentized Hall interval CI_{SH}.

For applied researchers the third evaluation criterion may be the most interesting one. A particular method is considered to work satisfactory if it is able to indicate the sign of the true underlying impulse response function with high probability. The corresponding results of our experiments are summarized in the last four columns of Table 2. Averaging over all horizons (first panel) we find that the asymptotic and the Hall interval do the best job in indicating the sign of the impulse response correctly for all but the relatively simple DGP (F), in which the Efron method is only marginally better than CI_A. Note that despite its good coverage properties, the Efron interval has often the lowest probability of indicating the correct sign. A possible explanation is that the Efron interval is located such that it includes the zero line, hence indicating no significant effect, while the underlying response is positive or negative. Moreover, we find that using the studentized Hall interval does indicate the correct signs less often than using the Hall percentile interval CI_H. This is probably due to its comparably long interval length.

A more disaggregated view on the results is presented in the last four panels of Table 2. Instead of averaging over all considered horizons h, we present average results for $h = 0, \dots, 12$, $h = 13, \dots, 24$, $h = 25, \dots, 36$ and $h = 37, \dots, 48$ to check whether our results are different for different horizons. Moving down the table (considering longer horizons), we find a tendency of increasing coverage rates, sometimes above the nominal level. This may be partly driven by the long-run zero restrictions on some response functions that also imply 'more precise' intervals but visual inspection of Figure 1 also suggests a similar effect for responses to permanent shocks. The relative performance of the intervals is, however, by and large similar when different h are considered. In particular, we still find that the Efron interval has the highest coverage. The relative length of the intervals only changes considerably for DGP (F) with different h: For short and medium horizons the asymptotic interval is about twelve times larger than the bootstrap interval, while it is only three times as wide for large h. Indicating the right sign is easier for all intervals when h is large. This is due to the fact that some of the responses are close to zero due to the long-run restrictions which is reflected relatively precisely by the intervals. Moreover, even responses to permanent shocks may have adjusted to their long-run values for large h which makes indicating their sign a lot easier. Nevertheless, CI_A and CI_H still do the best job in indicating the right sign.

A different view on our results is presented in Table 3, where we distinguish the results for responses to permanent and transitory shocks. This is instructive because permanent and transitory shocks have different implications on the shape of the response functions and, consequently, may affect the properties of the corresponding CIs. In Panel I of Table 3 we give the results derived for responses to permanent shocks. Note that CI_S still has average coverage rates closest to the nominal level for most DGPs. In some cases, however, the studentized Hall interval now has highest coverage rates. Moreover, for the permanent shocks CI_H and CI_{SH} feature somewhat higher coverage rates than CI_A which comes at the price of much wider intervals (an exception is DGPs (F)). Note, however, that CI_S indicates the right sign for many DGPs with only low probability (as low as 23%), i.e. despite its relative accurate coverage the Efron interval is not very informative about the underlying sign of responses to permanent shocks. Checking our simulation results more closely, we find that this is due to the relative broad bootstrap interval which is often located such that it includes the zero line. Indeed, the Efron interval indicates no significant effect while our true response is considered to have a positive effect on the variables in the system. While the Hall interval has the same length, it is obviously located differently, such that it also indicates the right sign more often. CI_A, CI_H and CI_{SH} perform very similarly with respect to the sign criterion. This result may be somewhat surprising given that CI_A is necessarily symmetric and the Hall intervals can in principle be asymmetric. We give the results for transitory shocks in Panel II of Table 3. Except for CI_{SH}, we now find a clear tendency for coverage rates to increase in comparison to results in Panel I. In particular, for CI_S we sometimes find higher than nominal coverage. Higher and more similar coverage rates are probably due to the long-run zero restrictions on the underlying impulse response functions. In contrast, the studentized Hall interval is much less precise in the case of transitory shocks (see also Figure 1). Interestingly, we also find comparable interval lengths (except for DGP (F)) and even cases where the bootstrap intervals CI_S and CI_H are smaller than the asymptotic interval. Note, however, that the studentized Hall interval is typically much longer than the asymptotic one.

In addition, we find that CI_S does usually the best job in indicating the response sign. This is not surprising given that the true sign of responses to transitory shocks is often 'zero' and given that the Efron interval contains the zero more often than the other intervals. We also note that differences between the strategies in terms of our sign criterion are usually smaller for transitory than for permanent shocks.

From the results in Table 3 we conclude that CI_A, CI_S, CI_H and CI_{SH} for responses to permanent shocks may be very different. In contrast, our results suggest that different intervals for responses to transitory shocks are much more alike.

In addition to the results discussed so far, we have also used non-normal errors (skewed and leptokurtic distributions as in Kilian (1998b)) for generating Monte Carlo time series. Although one would expect an advantage for the bootstrap method in this situation, the results are very similar to the ones given in Table 2 and 3. Consequently, they are not given here to conserve space.

4 Empirical Example

To illustrate the different CI properties discussed in the Monte Carlo comparison of Section 3, we apply all four CI construction methods in a structural VECM modeling U.S. macroeconomic data. In particular, we reconsider the three-variable SVECM first analyzed by King et al. (1991). The model we estimate is a cointegrated VAR for the log of output q_t, the log of consumption c_t and the log of investment i_t that was also the basis for DGP (A) in Section 3. In other words, $y_t = (q_t, c_t, i_t)'$.

We apply the usual two-step procedure for estimating the structural VECM. In a first step we estimate the reduced form VECM. In our case, we follow the original study in specifying the reduced form model. For this purpose we set the cointegration rank $r = 2$. The cointegration vectors are identified such that they may be interpreted as the great ratios between consumption, investment and income. Our reduced form model is the corresponding estimated VECM with 8 lags of Δy_t and an unrestricted constant. Secondly, we recover the structural shocks by imposing enough identifying restrictions. In this example with $K = 3$ variables and with $r = 2$ a maximum number of two shocks may have transitory effects (see discussion in Section 2). Consequently, there will be one permanent shock in this system. The permanent shock is identified by restricting the long-run effects of the last two structural shocks in the system to zero (as in the original paper by King et al. (1991)). In other words, the identified long-run impact matrix is given by

$$C(1)\mathsf{B} = \begin{pmatrix} * & 0 & 0 \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix},$$

where $*$ as before denotes unrestricted elements. In our framework, no further identifying assumptions are needed for the permanent shock (see also Breitung et al. (2004)). In contrast to the original paper we also identify the two transitory shocks for ML estimation of the structural parameters. To disentangle the two transitory shocks, we need to impose one additional restriction on the contemporaneous impact matrix B and for illustrative purposes we impose a recursive structure on the transitory shocks,

$$\mathsf{B} = \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & * & * \end{pmatrix},$$

i.e. the second transitory shock does not affect the second variable on impact. Using this identification scheme we estimate the structural parameters in B, the implied structural impulse response functions and the corresponding asymptotic (CI_A), Efron bootstrap (CI_S), Hall bootstrap (CI_H) and studentized Hall bootstrap (CI_{SH}) intervals with nominal coverage of 95%. CI_S, CI_H and CI_{SH} have been obtained using 1000 bootstrap draws. Moreover, we have used 100 inner bootstrap replications for constructing the studentized Hall interval.

The impulse response functions together with the three intervals (CI_A, CI_S, CI_H) are given in Figure 3. The first column shows the response of output, consumption and investment to the permanent shock. From Figure 3 we find that the three intervals in the first column differ substantially as we would have expected from our simulation results in Section 3. The symmetric asymptotic confidence band CI_A indicates a significant positive impact on all three variables. The Hall interval CI_H for the permanent shock is fairly asymmetric. In fact, the lower limit of the Hall interval is very similar to the lower limit of the asymptotic interval. However, the upper limit is much bigger than the upper limit of CI_A. Consequently, the bootstrap interval is much wider which illustrates a typical result from our simulation study. Our example also illustrates that despite fairly different interval length, CI_A and CI_H suggest at all horizons the same sign for the response to the permanent shock precisely because of the asymmetry of CI_H. Judging significance according to the Efron interval CI_S leads to completely different conclusions. Note that the upper limit of CI_S is very similar to the lower limit of CI_A. However, due to the asymmetry of CI_S, the zero line is included in all intervals and consequently no significant responses can be diagnosed. This illustrates another simulation result, namely that the Efron CIs contain the zero line more often than the other methods and consequently are less informative with respect to the sign of the impulse responses.

Visual inspection of the responses to transitory shocks reveals that the three intervals are much more alike than in the case of permanent shocks. In particular, we find that for $h > 8$ all three intervals allow the same interpretation of the impulse responses. However, as in the case of permanent shocks, the Efron interval CI_S includes the zero line almost always, hence being again not very informative with respect to the sign.

Figure 4 shows a comparison between the Hall percentile interval (CI_H) and the studentized Hall interval (CI_{SH}). It is evident from that figure that the studentized Hall interval is wider and much more asymmetric than CI_H. In line with the results from Figure 3, also the two Hall intervals are much more alike for the two transitory shocks.

5 Concluding Remarks

We have investigated the finite sample properties of different confidence interval construction methods in the framework of structural vector error correction models (SVECMs) by means of a Monte Carlo study. We have focused on the comparison of four methods that have been often used in practice for constructing SVECM impulse response CIs but whose finite sample properties have been unknown to date. In the Monte Carlo design, we have included DGPs that are based on empirical SVECM studies. Therefore, we have covered a wide range of models with typical size, dynamics and identification schemes. We find that all methods produce coverage rates slightly below the nominal level for short response horizons. In addition, we also find a tendency of increasing coverage at larger horizons. This suggests that all intervals are more accurate at medium and long-run horizons.

Our comparison of alternative methods leads to different results for responses to permanent and transitory shocks: If responses to permanent shocks are analyzed, the four considered methods may lead to substantially different confidence intervals. We find that asymptotic and Hall percentile bootstrap intervals have similar coverage rates. Although the former interval is usually asymmetric and much wider, both indicate the right sign of the underlying response equally often and allow similar interpretations. In contrast, despite its relatively good coverage properties the Efron bootstrap interval includes the zero line more often and consequently indicates the correct sign less often. Moreover, according to our simulation results the studentized Hall bootstrap interval typically does not show its theoretical advantages in terms of coverage, interval length and its ability to indicate the right sign. Interestingly, the intervals for responses to transitory shocks are much more alike. All methods (with the exception of the studentized Hall interval) have similar coverage properties and produce intervals of comparable length in this case. We have presented an empirical example based on a three-dimensional SVECM for U.S. data which illustrates our main findings from the simulation study.

Based on our Monte Carlo evidence, it appears that applied researchers have little to choose between the asymptotic and the Hall bootstrap intervals in SVECMs. In contrast, the Efron bootstrap interval may be less suitable for applied work as it is less informative about the sign of the underlying impulse response function. Moreover, the computationally demanding studentized Hall interval does not show its theoretical advantage and is often outperformed by the other methods.

An interesting result is the remarkable accuracy of asymptotic interval even in models with many variables and lags. Even if the error distribution is skewed or leptokurtic, gains from using bootstrap methods are apparently very limited. These results are in contrast to results from previous studies for unrestricted VARs by e.g. Kilian (1998b) who finds that bootstrap methods usually produce much more accurate confidence intervals.

References

Amisano, G. & Giannini, C. (1997). *Topics in Structural VAR Econometrics*, 2nd edn, Springer–Verlag.

Benkwitz, A., Lütkepohl, H. & Wolters, J. (2001). Comparison of bootstrap confidence intervals for impulse responses of German monetary systems, *Macroeconomic Dynamics* **5**: 81–100.

Benkwitz, A., Lütkepohl, H. & Neumann, M. H. (2000). Problems related to bootstrapping impulse responses of autoregressive processes, *Econometric Reviews* **19**: 69–103.

Breitung, J., Brüggemann, R. & Lütkepohl, H. (2004). Structural vector autoregressive modeling and impulse responses, *in* H. Lütkepohl & M. Krätzig (eds), *Applied Time Series Econometrics*, Cambridge University Press, Cambridge, pp. 159–196.

Brüggemann, R. (2004). *Model Reduction Methods for Vector Autoregressive Processes*, Vol. 536 of *Lecture Notes in Economics and Mathematical Systems*, Berlin: Springer-Verlag.

Coenen, G. & Vega, J. L. (1999). The demand for M3 in the Euro area, *Working Paper No. 6*, European Central Bank.

Coenen, G. & Vega, J. L. (2001). The demand for M3 in the Euro area, *Journal of Applied Econometrics* **16**: 727–748.

Efron, D. & Tibshirani, R. (1993). *An Introduction to the Bootstrap*, Chapman & Hall.

Fisher, L. A., Fackler, P. L. & Orden, D. (1995). Long-run identifying restrictions for an error-correction model of New Zealand money, prices and output, *Journal of International Money and Finance* **14**(1): 127–147.

Fisher, L. A., Huh, H. & Tallman, E. W. (2003). Permanent income and transitory variation in investment and output, *Journal of Macroeconomics* **25**: 149–168.

Gonzalo, J. & Ng, S. (2001). A systematic framework for analyzing the dynamic effects of permanent and transitory shocks, *Journal of Economic Dynamics and Control* **25**: 1527–1546.

Hall, P. (1992). *The Bootstrap and Edgeworth Expansion*, New York: Springer.

Jang, K. & Ogaki, M. (2004). The effects of monetary policy shocks on exchange rates: a structural vector error correction model approach, *Journal of the Japanese and International Economies* **18**: 99–114.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autogressive Models*, Oxford University Press.

Kilian, L. (1998a). Accounting for lag order uncertainty in autoregressions: The endogenous lag order bootstrap algorithm, *Journal of Time Series Analysis* **19**(5): 531–548.

Kilian, L. (1998b). Confidence intervals for impulse responses under departures from normality, *Econometric Reviews* **17**(1): 1–29.

Kilian, L. (1998c). Small sample confidence intervals for impulse response functions, *Review of Economics and Statistics* **80**(2): 218–230.

Kilian, L. (2001). Impulse response analysis in vector autoregressions with unknown lag order, *Journal of Forecasting* **20**: 161–179.

Kilian, L. & Chang, P.-L. (2000). How accurate are confidence intervals for impulse responses in large VAR models?, *Economics Letters* **69**: 299–307.

King, R. G., Plosser, C. I., Stock, J. H. & Watson, M. (1991). Stochastic trends and economic fluctuations, *American Economic Review* **81**(4): 819–840.

Levtchenkova, S., Pagan, A. R. & Robertson, J. (1998). Shocking stories, *Journal of Economic Surveys* **12**: 507–532.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Berlin: Springer-Verlag.

Lütkepohl, H. & Krätzig, M. (eds) (2004). *Applied Time Series Econometrics*, Cambridge University Press, Cambridge.

Lütkepohl, H. & Reimers, H.-E. (1992). Impulse response analysis of cointegrated systems, *Journal of Economic Dynamics and Control* **16**: 53–78.

Lütkepohl, H. & Wolters, J. (2003). Transmission of German monetary policy in the pre-euro period, *Macroeconomic Dynamics* **7**: 711–733.

Mellander, E., Vredin, A. & Warne, A. (1992). Stochastic trends and economic fluctuations in a small open economy, *Journal of Applied Econometrics* **7**: 369–394.

Phillips, P. C. B. (1998). Impulse response and forecast error variance asymptotics in nonstationary VARs, *Journal of Econometrics* **83**: 21–56.

Ribba, A. (2003a). Permanent-transitory decompositions and traditional measures of core inflation, *Economics Letters* **81**: 109–116.

Ribba, A. (2003b). Short-run and long-run interaction between inflation and unemployment in the USA, *Applied Economics Letters* **10**: 373–376.

Sims, C. A. & Zha, T. (1999). Error bands for impulse responses, *Econometrica* **67**(5): 1113–1155.

Vlaar, P. J. G. (2004a). On the asymptotic distribution of impulse response functions with long-run restrictions, *Econometric Theory* **20**(5): 891–903.

Vlaar, P. J. G. (2004b). Shocking the eurozone, *European Economic Review* **48**: 109–131.

Table 1: DGPs based on empirical VECMs

	K	r	p		moduli[a]
DGP (A):	3	2	9	$y_t' = (c_t, i_t, q_t),\ \ \beta' = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$	1.00; 0.93; 0.90; 0.84; 0.82; 0.81; 0.80; 0.79; 0.70; 0.68; 0.66; 0.65; 0.54
Structural identification: $C(1)\mathsf{B} = \begin{pmatrix} * & 0 & 0 \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix},\ \mathsf{B} = \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & * & * \end{pmatrix}$, Ref.: King et al. (1991)					
DGP (B): see DGP (A) but with $p = 2$.					1.00; 0.90; 0.76; 0.32; 0.28;
DGP (C):	6	3	9	$y_t' = (c_t, i_t, m_t, q_t, R, \Delta p_t),$ $\beta' = \begin{pmatrix} 1 & 0 & 0 & -1.13 & 0.003 & 0.009 \\ 0 & 1 & 0 & -1.06 & -0.001 & 0.005 \\ 0 & 0 & 1 & -0.97 & 0.016 & -0.031 \end{pmatrix}$	1.00; 1.00; 1.00; 0.94; 0.92; 0.91; 0.90; 0.89; 0.88; 0.87; 0.84; 0.83; 0.82; 0.81; 0.78; 0.79; 0.77; 0.66; 0.09;
Structural identification: $C(1)\mathsf{B} = \begin{pmatrix} * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \end{pmatrix},\ \mathsf{B} = \begin{pmatrix} * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{pmatrix}$, Ref.: King et al. (1991)					
DGP (D): see DGP (C) but with $p = 2$.					1.00; 1.00; 1.00; 0.89; 0.75; 0.52; 0.39; 0.34; 0.18; 0.02;
DGP (E):	5	3	2	$y_t' = (m3_t, \pi_t, R_t^s, y_t, R^l),$ $\beta' = \begin{pmatrix} 1 & 0 & 0 & -1.36 & 0.10 \\ 0 & 1 & 0 & 0.07 & -0.51 \\ 0 & 0 & 1 & 0.03 & -0.77 \end{pmatrix}$	1.00; 0.92; 0.70; 0.57 0.36; 0.28;
Structural identification: $C(1)\mathsf{B} = \begin{pmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \end{pmatrix},\ \mathsf{B} = \begin{pmatrix} * & * & * & 0 & 0 \\ * & * & * & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$, Ref.: Coenen & Vega (2001)					
DGP (F):	2	1	7	$y_t' = (\pi_t, U_t),\ \ \beta' = (1, -1)$	1.00; 0.92; 0.76; 0.74 0.72; 0.66; 0.63
Structural identification: $C(1)\mathsf{B} = \begin{pmatrix} * & 0 \\ * & 0 \end{pmatrix},\ \mathsf{B} = \begin{pmatrix} * & * \\ * & * \end{pmatrix}$, Ref.: Ribba (2003b)					

Note: The remaining VECM parameters for DGPs (A) to (F) are obtained by estimating a VECM with $p-1$ lags of Δy_t for given β. An unrestricted constant is included in all models. [a]Entries are the moduli of the nonzero eigenvalues of the respective VAR companion matrix. Structural identification: $*$ denotes an unrestricted element.

Table 2: Average coverage rate, length and relative frequency of correct implied sign for nominal 90% asymptotic and bootstrap confidence intervals

		Coverage EC				Length AL		Sign CS			
DGP	h	CI_A	CI_S	CI_H	CI_{SH}	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{H}}$	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{SH}}$	CI_A	CI_S	CI_H	CI_{SH}
(A)	0 - 48	0.859	0.896	0.841	0.816	0.942	0.915	0.885	0.794	0.839	0.812
(B)	0 - 48	0.750	0.862	0.685	0.677	0.906	0.536	0.865	0.710	0.808	0.752
(C)	0 - 48	0.732	0.823	0.729	0.714	0.815	0.789	0.776	0.765	0.734	0.710
(D)	0 - 48	0.573	0.644	0.569	0.507	0.902	0.631	0.832	0.738	0.799	0.751
(E)	0 - 48	0.680	0.727	0.665	0.619	0.858	0.715	0.837	0.802	0.763	0.658
(F)	0 - 48	0.847	0.844	0.843	0.893	5.215	0.631	0.931	0.947	0.882	0.773
(A)	0 - 12	0.796	0.814	0.730	0.754	1.008	0.847	0.719	0.666	0.678	0.677
(B)	0 - 12	0.780	0.854	0.681	0.679	0.946	0.803	0.754	0.665	0.664	0.631
(C)	0 - 12	0.615	0.722	0.622	0.645	0.808	0.697	0.661	0.765	0.621	0.616
(D)	0 - 12	0.665	0.727	0.605	0.569	0.864	0.827	0.697	0.616	0.647	0.603
(E)	0 - 12	0.665	0.716	0.665	0.634	0.791	0.711	0.741	0.691	0.670	0.626
(F)	0 - 12	0.857	0.918	0.849	0.926	12.505	0.523	0.833	0.833	0.611	0.364
(A)	13 - 24	0.872	0.920	0.829	0.800	0.976	0.923	0.888	0.818	0.802	0.775
(B)	13 - 24	0.784	0.885	0.726	0.676	0.882	0.672	0.901	0.725	0.819	0.718
(C)	13 - 24	0.747	0.860	0.731	0.711	0.813	0.774	0.774	0.780	0.724	0.698
(D)	13 - 24	0.620	0.672	0.601	0.520	0.879	0.767	0.854	0.755	0.807	0.728
(E)	13 - 24	0.681	0.736	0.649	0.614	0.870	0.742	0.806	0.841	0.700	0.601
(F)	13 - 24	0.823	0.873	0.788	0.915	12.266	0.514	0.850	0.868	0.649	0.236
(A)	25 - 36	0.883	0.924	0.892	0.840	0.914	0.942	0.965	0.848	0.924	0.878
(B)	25 - 36	0.736	0.863	0.679	0.673	0.898	0.428	0.908	0.728	0.872	0.810
(C)	25 - 36	0.782	0.865	0.775	0.739	0.824	0.831	0.829	0.806	0.781	0.747
(D)	25 - 36	0.532	0.608	0.555	0.479	0.905	0.539	0.891	0.792	0.869	0.824
(E)	25 - 36	0.686	0.728	0.670	0.612	0.889	0.724	0.889	0.840	0.810	0.668
(F)	25 - 36	0.800	0.818	0.763	0.888	7.524	0.570	0.853	0.868	0.858	0.691
(A)	37 - 48	0.889	0.933	0.923	0.875	0.865	0.950	0.979	0.854	0.964	0.930
(B)	37 - 48	0.698	0.845	0.654	0.682	0.895	0.240	0.908	0.728	0.889	0.860
(C)	37 - 48	0.794	0.852	0.799	0.764	0.815	0.860	0.852	0.806	0.819	0.786
(D)	37 - 48	0.466	0.562	0.514	0.457	0.963	0.373	0.897	0.799	0.885	0.863
(E)	37 - 48	0.687	0.728	0.694	0.615	0.889	0.682	0.919	0.846	0.880	0.741
(F)	37 - 48	0.830	0.856	0.823	0.880	3.402	0.418	0.959	0.977	0.949	0.842

Note: CI_A, CI_S, CI_H and CI_{SH} denote the asymptotic, the standard percentile interval, the Hall percentile and the studentized Hall interval (see Sections 2.1 and 2.2). $\mathsf{AL}_A/\mathsf{AL}_\mathsf{H}$ and $\mathsf{AL}_A/\mathsf{AL}_\mathsf{SH}$ are the relative average interval lengths. Results are based on DGPs (A) to (F) given in Table 1. Table entries are based on averaging the respective quantity over all response functions in the system and over horizons h given in the second column.

Table 3: Average coverage rate, length and relative frequency of correct implied sign for nominal 90% asymptotic and bootstrap confidence intervals: permanent vs. transitory shocks

Panel I: Permanent Shocks

		Coverage EC				Length AL		Sign CS			
DGP	h	CI_A	CI_S	CI_H	CI_{SH}	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{H}}$	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{SH}}$	CI_A	CI_S	CI_H	CI_{SH}
(A)	0 - 48	0.762	0.802	0.833	0.862	0.665	0.554	0.870	0.539	0.872	0.872
(B)	0 - 48	0.679	0.906	0.680	0.702	0.310	0.235	0.719	0.228	0.711	0.708
(C)	0 - 48	0.570	0.693	0.616	0.630	0.711	0.597	0.637	0.549	0.620	0.619
(D)	0 - 48	0.561	0.647	0.576	0.572	0.582	0.527	0.731	0.510	0.730	0.728
(E)	0 - 48	0.596	0.628	0.628	0.649	0.745	0.578	0.762	0.602	0.735	0.701
(F)	0 - 48	0.842	0.880	0.820	0.911	9.921	0.638	0.891	0.899	0.898	0.542

Panel II: Transitory Shocks

		Coverage EC				Length AL		Sign CS			
DGP	h	CI_A	CI_S	CI_H	CI_{SH}	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{H}}$	$\frac{\mathsf{AL}_A}{\mathsf{AL}_\mathsf{SH}}$	CI_A	CI_S	CI_H	CI_{SH}
(A)	0 - 48	0.907	0.943	0.846	0.793	1.081	1.097	0.892	0.921	0.822	0.793
(B)	0 - 48	0.786	0.839	0.688	0.665	1.204	0.692	0.938	0.952	0.857	0.774
(C)	0 - 48	0.848	0.916	0.811	0.773	0.889	0.926	0.876	0.919	0.816	0.775
(D)	0 - 48	0.581	0.642	0.565	0.462	1.130	0.705	0.904	0.901	0.847	0.768
(E)	0 - 48	0.727	0.782	0.686	0.602	0.922	0.791	0.879	0.914	0.779	0.634
(F)	0 - 48	0.814	0.855	0.793	0.895	8.074	0.375	0.856	0.873	0.629	0.518

Note: CI_A, CI_S, CI_H and CI_{SH} denote the asymptotic, the standard percentile interval, the Hall percentile and the studentized Hall interval (see Sections 2.1 and 2.2). $\mathsf{AL}_A/\mathsf{AL}_\mathsf{H}$ and $\mathsf{AL}_A/\mathsf{AL}_\mathsf{SH}$ are the relative average interval lengths. Results are based on DGPs (A) to (F) given in Table 1. Entries in Panel I are obtained by averaging the respective quantity over all response functions to permanent shocks in the system and over horizons h given in the second column. Accordingly, Panel II gives the results from averaging over results for transitory shocks.

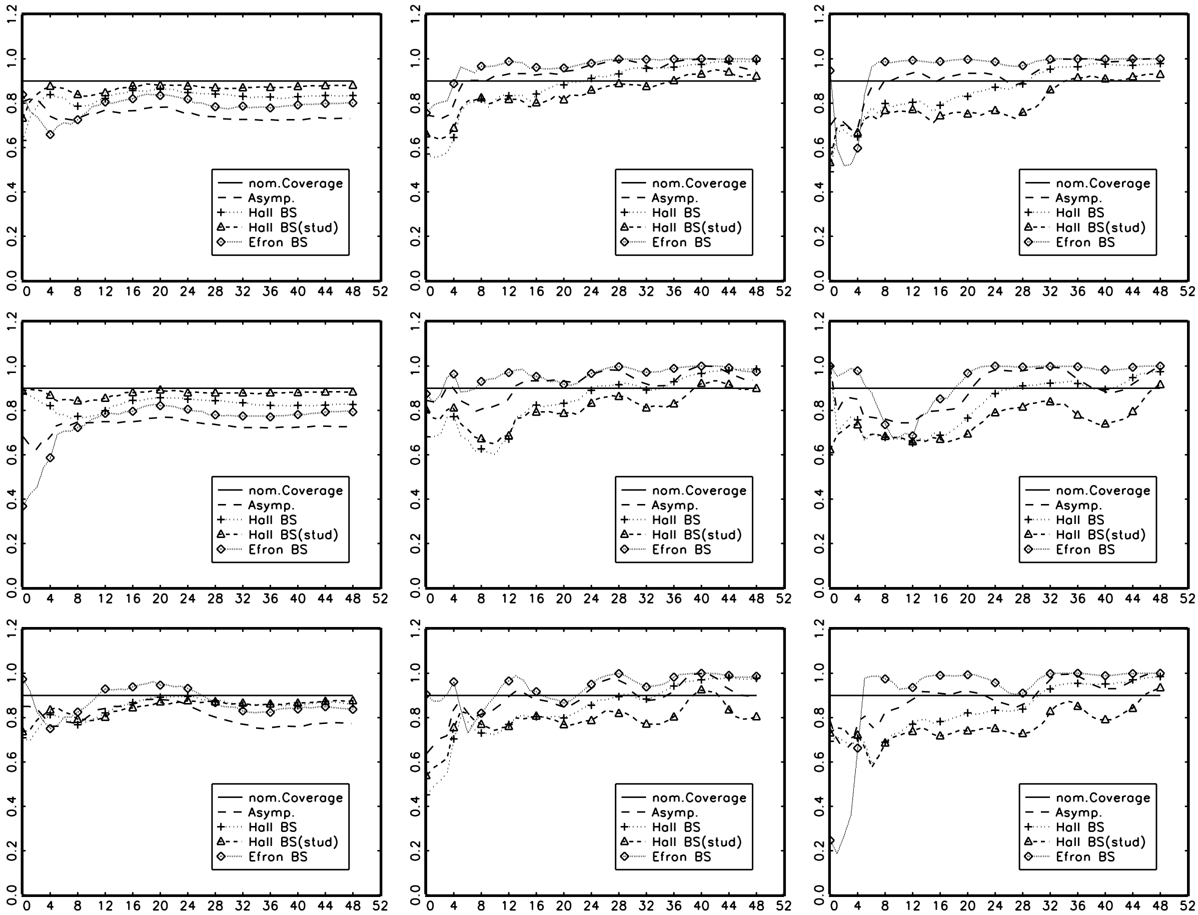

Figure 1: Empirical coverage of different impulse response intervals. Results for DGP (A) from Table 1 based on 1000 Monte Carlo replications with 500 bootstrap draws each. 100 inner bootstrap replications have been used for the studentized Hall interval. Nominal coverage is 90%.

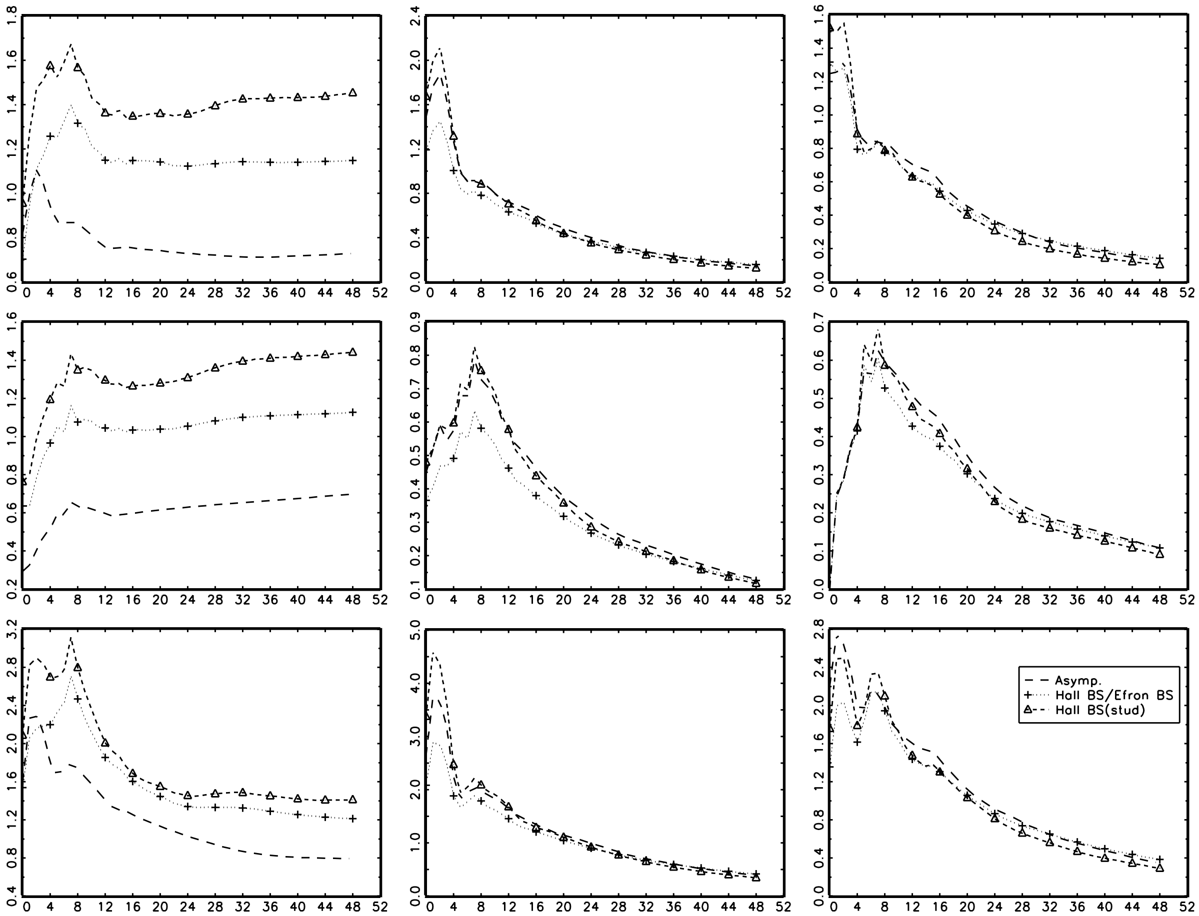

Figure 2: Average length of different impulse response intervals. Results for DGP (A) from Table 1 based on 1000 Monte Carlo replications with 500 bootstrap draws each. 100 inner bootstrap replications have been used for the studentized Hall interval. Nominal coverage is 90%.

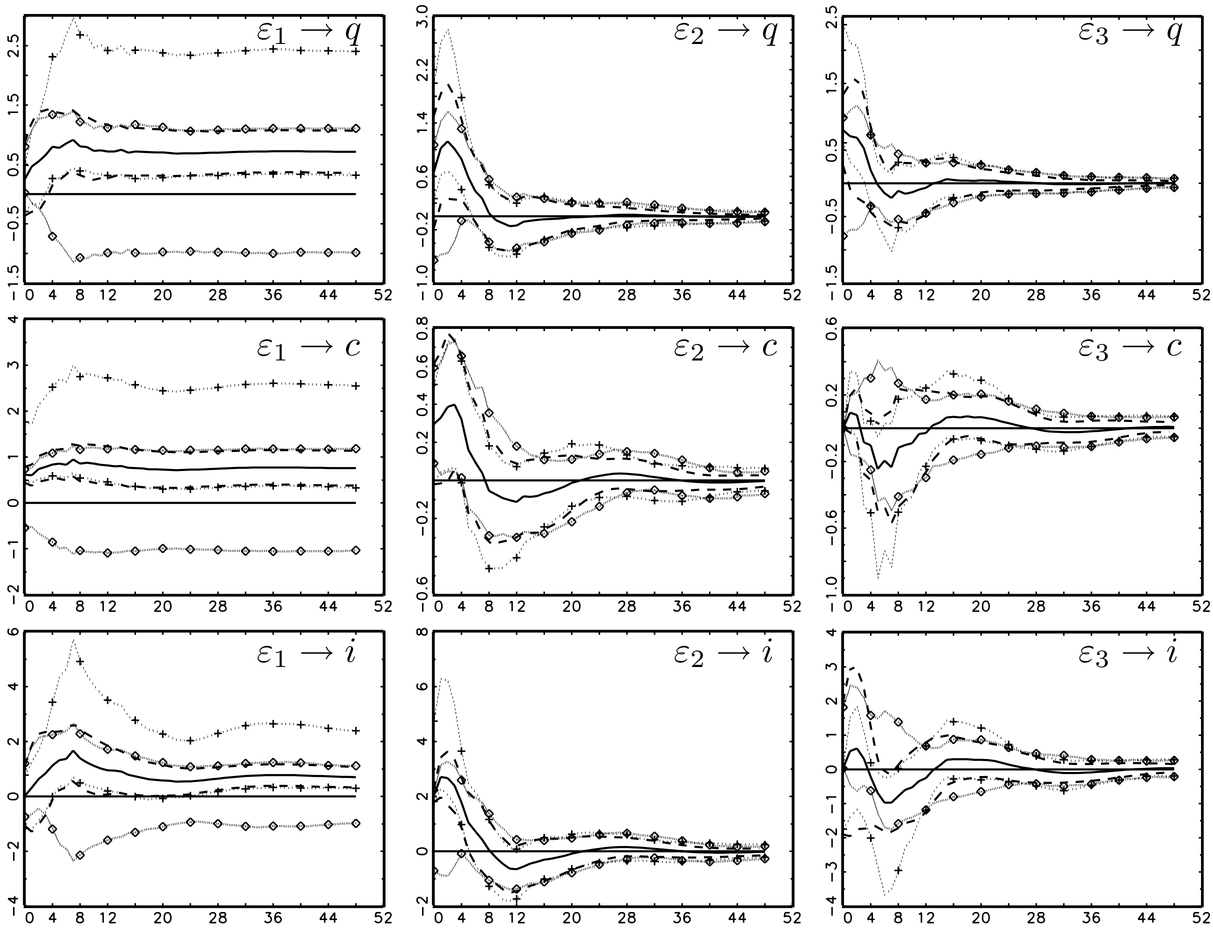

Figure 3: 95% impulse response intervals in SVECM for the log output q_t, the log of consumption c_t and the log of investment i_t. Point estimate (—), asymptotic interval CI_A (- - -), Hall bootstrap interval CI_H ($+ \cdots + \cdots +$) and Efron bootstrap interval CI_S ($\Diamond \cdots \Diamond \cdots \Diamond$). The bootstrap intervals are based on 1000 bootstrap replications.

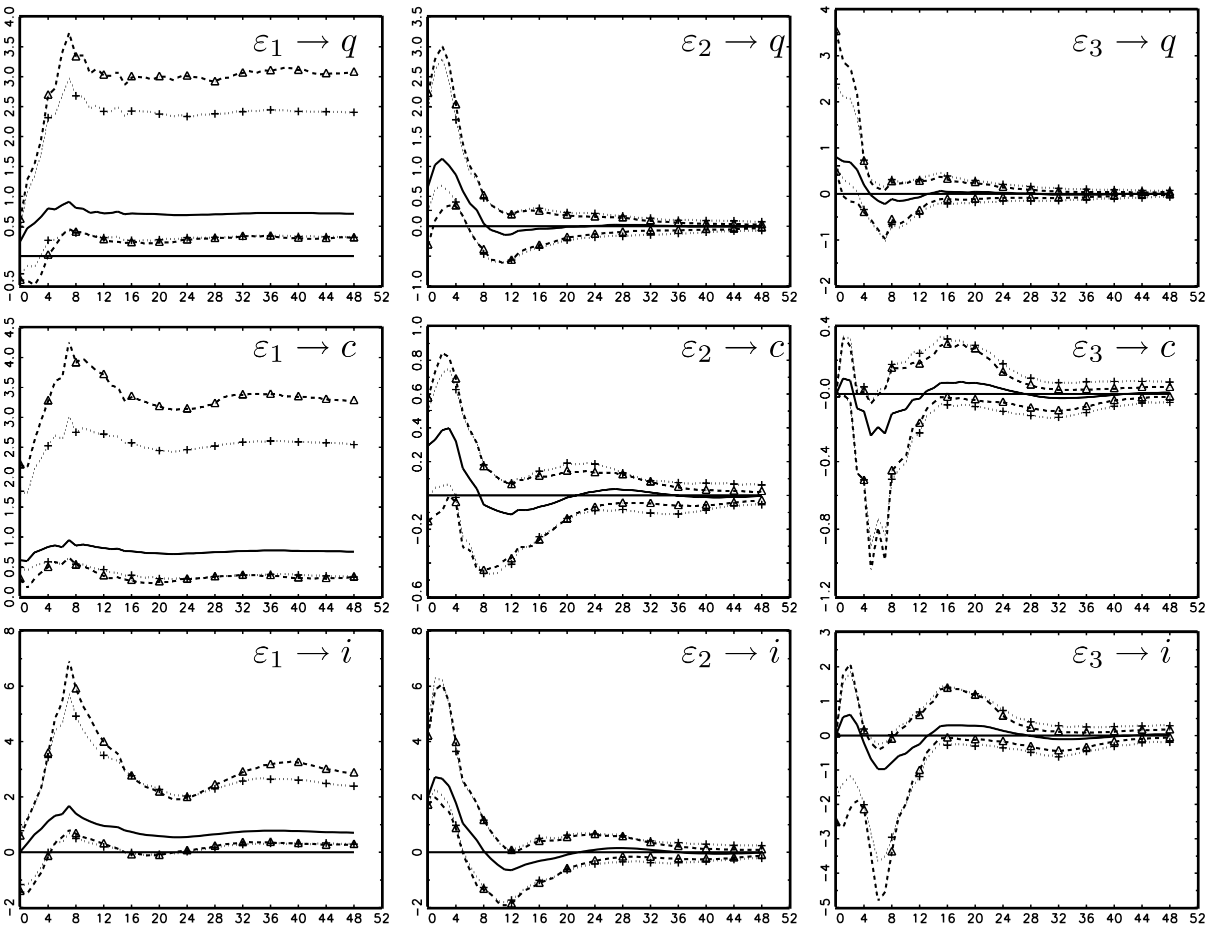

Figure 4: 95% impulse response intervals in SVECM for the log output q_t, the log of consumption c_t and the log of investment i_t. Point estimate (—), Hall bootstrap interval CI_H ($+\cdots+\cdots+$) and studentized Hall bootstrap interval CI_{SH} ($\triangle--\triangle--\triangle$). The bootstrap intervals based on 1000 bootstrap replications. 100 inner bootstrap replications have been used for the studentized Hall interval.

A Asymptotic Distribution of Structural Impulse Responses

The structural impulse response coefficients defined in Section 2.1 are given by

$$\Theta_i = \Phi_i \mathsf{B}. \tag{A.1}$$

The corresponding estimated quantities are asymptotically normal as they are nonlinear functions of asymptotically normal parameter estimators. More precisely,

$$\sqrt{T}\text{vec}(\hat{\Theta}_i - \Theta_i) \to N(0, \Sigma_{\Theta_i}). \tag{A.2}$$

Vlaar (2004a) gives an explicit expression for the covariance matrix. Let $\Sigma_\gamma = \Sigma_{\Delta Y}^{-1} \otimes \Sigma_u$ be the covariance matrix of the VEC parameters, where $\Sigma_{\Delta Y} = \text{plim } ZZ'/T$ with the definition of $Z_t := (\Delta y_t', \dots, \Delta y_{t-p+2}', \beta' y_t)$ and $Z := (Z_0, \dots, Z_{T-1})'$. Moreover, let $\Sigma_{\gamma_\mathsf{B}}$ be the covariance matrix of the free structural parameters γ_B, which can be computed as the inverse of the information matrix of γ_B in (2.3) (see also Amisano & Giannini (1997) for precise expressions). Then the covariance matrix in (2.8) has the following structure.

$$\Sigma_{\Theta_i} = (F_i + G_i)\Sigma_\gamma (F_i + G_i)' + H_i \Sigma_{\gamma_\mathsf{B}} H_i', \tag{A.3}$$

with $F_0 = 0$,

$$F_i = \sum_{m=0}^{i-1} \mathsf{B}' J (\mathbf{A}')^{i-1-m} W \otimes J \mathbf{A}^m J', \qquad i = 1, 2, \dots, \tag{A.4}$$

where $J = (I_K, 0, \dots, 0)$ is a $K \times Kp$ matrix,

$$\mathbf{A} := \begin{pmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_K & 0 & \dots & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_K & 0 \end{pmatrix}$$

is the companion matrix of the VAR in levels corresponding to the VECM and W is the matrix that relates the VAR parameters to the VEC parameters defined as

$$W := \begin{pmatrix} I_K & 0 & \cdots & 0 & 0 & I_K \\ -I_K & I_K & \ddots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & -I_K & I_K & 0 \\ 0 & 0 & 0 & 0 & -I_K & 0 \end{pmatrix} \begin{pmatrix} I_{K(p-1)} & 0 \\ 0 & \beta \end{pmatrix}.$$

Furthermore,

$$H_i = (I_K \otimes J \mathbf{A}^i J') R_\perp, \qquad i = 0, 1, \dots, \tag{A.5}$$

where $R_\perp$ is an orthogonal complement of the restriction matrix $R = [R_l^{*\prime}, R_s^\prime]^\prime$ and

$$G_i = (I_K \otimes J\mathbf{A}^i J')\bar{G}, \qquad i = 0, 1, \dots, \tag{A.6}$$

where $\bar{G}$ is a complicated expression that involves the extra derivatives due to the stochastic nature of the restriction matrix R. An explicit expression of $\bar{G}$ is derived by Vlaar (2004a) and is given here for completeness:

$$\begin{aligned}\bar{G} = &- (I_{K^2} - R_\perp'(R_\perp(\mathsf{B}' \otimes I_K)(I_{K^2} + \mathbf{K}_{KK})(\mathsf{B} \otimes I_K)R_\perp')^{-1}R_\perp(\mathsf{B}' \otimes I_K) \\ &\times (I_{K^2} + \mathbf{K}_{KK})(\mathsf{B} \otimes I_K))(\text{vec}(\mathsf{B})' \otimes R^+) \\ &\times \left(I_{K^2} \otimes \begin{pmatrix} I_l \\ 0_{s\times l}\end{pmatrix} R_l\right)\begin{pmatrix} I_{K^3} & 0_{K^3\times K}\end{pmatrix} \\ &\times \left(\mathbf{1}_K \otimes \begin{bmatrix} 1 & 0_K & & 0_{K(K-1)} \\ 0_{K^2} & 1 & \ddots & 1 \\ & 0_{K(K-1)} & & 0_K \end{bmatrix}\right) \otimes I_K \\ &\times \left[(\mathbf{1}_{p-1}' \otimes C(1)'),\ (C(1)'\Psi' - I_K)\alpha(\alpha'\alpha)^{-1}\right] \otimes C(1),\end{aligned} \tag{A.7}$$

where $\mathbf{K}_{KK}$ is a $K^2 \times K^2$ commutation matrix defined such that for any $K \times K$ matrix M, $\text{vec}(M') = \mathbf{K}_{KK}\text{vec}(M)$, R^+ is the Moore-Penrose inverse of R and $\mathbf{1}_K$ denotes a K-dimensional vector of ones. In addition, l and s denote the number of rows of R_l and R_s, respectively.

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