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Volatility forecasting for low-volatility investing

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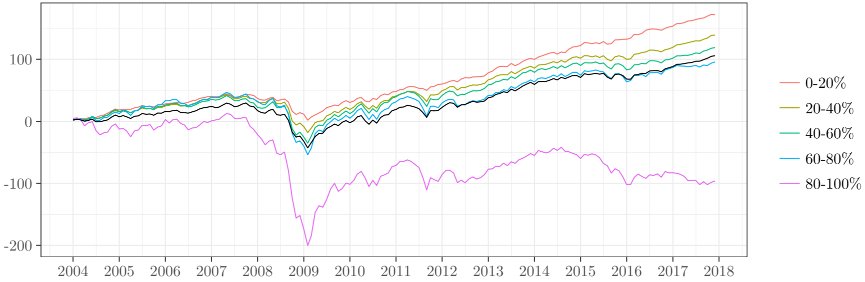
Cumulative excess returns by volatility quintile - S&P 500



Equally-weighted portfolios are formed each month based on ex-post "oracle" (this-month's) volatility. Fama-French excess market return in black

In the following, low-volatility portfolios are defined to include stocks that are in the lowest cross-sectional volatility quintile

Anomaly dates back to Haugen and Heins (1972); more literature: Ang et al. (2006, 2009); Blitz and van Vliet (2007); Baker et al. (2011); Driessen et al. (2017)

Agenda

We ask whether the **low-volatiltiy anomaly** is exploitable by

- making use of high-frequency based measures of realized variation
- employing recent advances in forecast evaluation
- in order to choose among a large number of models for > 500 stocks
- and form portfolios based on these forecasts

We do not focus on

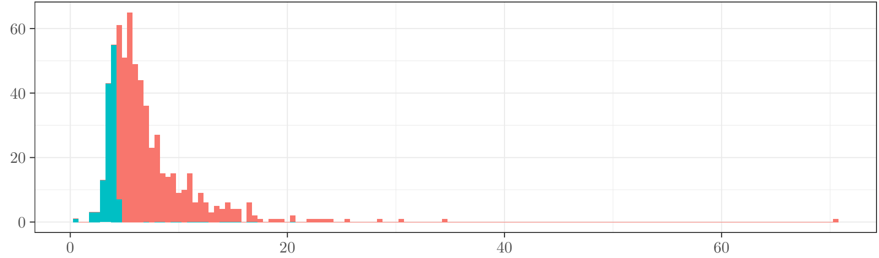
- investing in minimum-variance portfolios
- modeling high-dimensional covariance matrices

Background:

- Low-volatility funds use predominantly daily stock returns (last-month sq. ret.) which is at odds with the volatility forecasting literature
- Side question: Is the low-volatility anomaly more/less pronounced for better volatility proxies?

What is the best way to form low-volatility portfolios ex-ante?

Cross-sectional volatility in May 2017



We need "good" forecasts!

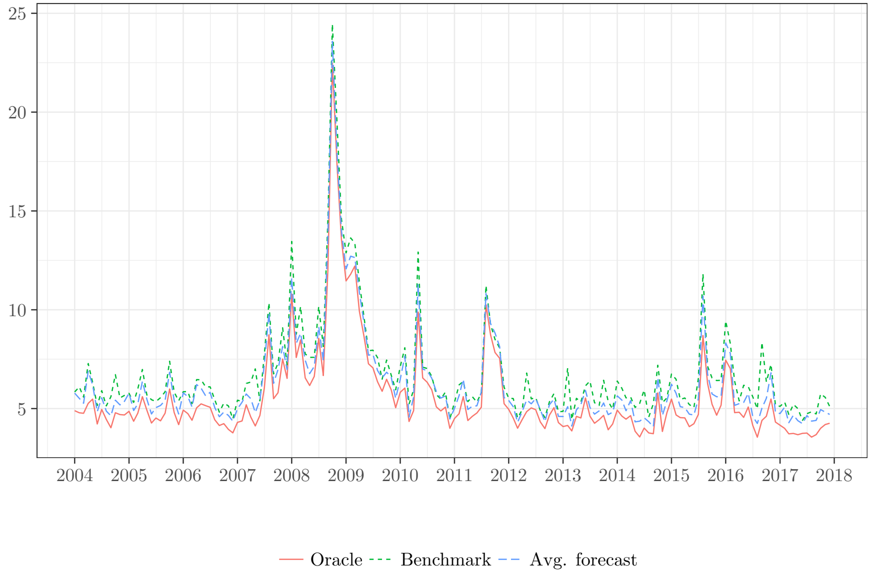
- But what is "good" in terms of our portfolio choice problem?
  $\Rightarrow$ Ranking vs. level of volatility
- There are many time series models out there - possibly different performance across stocks and time!?

Preliminary results

- Low-volatility investing may not be low-volatility investing
- Portfolio choices based on time series models are more “stable”
- They outperform benchmark case after considering transaction costs
- Avg. forecast seems to strike a good balance for equal-weighted portfolios
- Forecast accuracy itself is not a good way for choosing models

Model	Oracle-overlap	ARV^o	Turnover	Return
Oracle	–	36.75	0.71	12.28
Avg. forecast	0.67	45.09	0.34	8.83
Elem. score (best)	0.67	45.79	0.44	9.05
Benchmark	0.59	52.46	0.94	8.54

Average volatility for three portfolio sorts



Our results in more detail

- Bregman loss functions and portfolio sorts

- Low-volatility investing
  - Data
  - Models
  - Univariate equally-weighted portfolio sorts

Forecast evaluation for portfolios (I)

Let x denote a volatility forecast and y the corresponding realization.

- Most volatility time-series models are built to forecast the conditional mean
- Forecast evaluation by SE $[(x-y)^2]$ or QLIKE $[y/x - log(y/x) - 1]$?
- We are not so much interested in the actual values of forecast errors if we sort in line with the low-volatility cut-off of realized volatility (for now)
- Additionally, the Bregman class of loss functions for which the conditional mean is "optimal" is large:

$$s(x,y) = \phi(y) - \phi(x) - \phi'(x)(y-x)$$

where ϕ is a convex function with subgradient ϕ'

Forecast evaluation for portfolios (II)

- Elementary score (EL): Ehm *et al.* (2016) showed that each Bregman loss function can be written as

$$\int_{\theta=0}^{\infty} EL_\theta(x, y) dH(\theta)$$

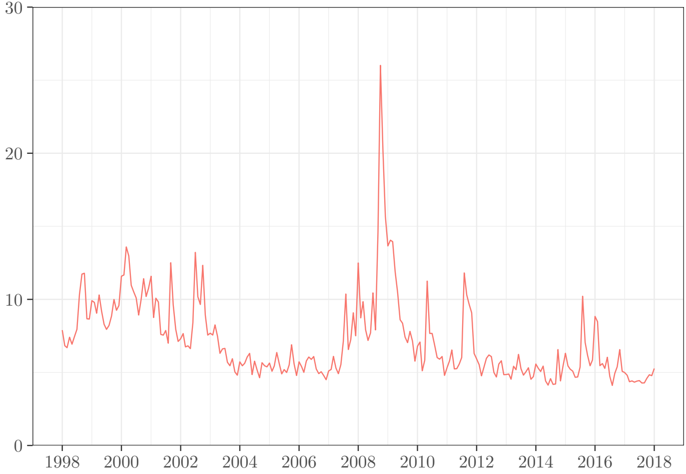
with

$$EL_\theta(x, y) = \begin{cases} |y - \theta| & \text{if } \min(x, y) \leq \theta < \max(x, y) \\ 0 & \text{else} \end{cases}$$

and H being a positive weighting function

- For each θ, EL_θ assigns a penalty in terms of absolute error if sorting went "wrong"
- This is equivalent to our portfolio choice problem if the "true" θ would be known
- However, we discard a lot of valuable information

Monthly cross-sectional 20% RV^o quantile



Obviously, low-volatility cut-off is time-varying

Data

Prices for all current and former S&P 500 stocks (obtained from QuantQuote)

- 1998M1 - 2017M12, one-minute prices
- 1026/842 stocks, 5064 daily observations
- $RV_t^d =$ sq. daily ret.
- $RV_t^o = RV +$ sq. overnight (sub-sampled)
- $medRV$, RQ
- Semi-variances: RV^+, RV^-, SJV

French:

- Market return
- Risk-free rate
- FFC factors

Cboe: VIX

Rolling window estimation and evaluation (4+2 years)

168 monthly portfolio returns, 2004M1 – 2017M12

Models

At the end of each month we calculate

- Random-walk forecast $RV^o_{t+1:t+22|t} = RV^o_{t-21:t}$ and $RV^d_{t+1:t+22|t} = RV^d_{t-21:t}$
- Autoregressive model based on rank statistic
- Idiosyncratic volatility (Fama-French three-factor)
- Exponentially weighted moving average on monthly aggregates (Riskmetrics), $\lambda = 0.98$
- Different HAR forecasts

$$RV^o_{t+1:t+22|t} = \beta_0 + \beta_d RV_t + \beta_w RV_{t-4:t} + \beta_m RV^o_{t-21:t} (+ \dots)$$

  and variants with jumps, VIX, RV in logs, ...
- For comparison: two oracle forecasts

Portfolio sorts

- Single model for all stocks: Form portfolios by forecast-implied volatility ranking
- Same for average forecast (based on HAR-like models)
- Loss function: For each stock, calculate the out-of-sample loss over the last two years. Choose the “best” model for each stock accordingly. We use RV_t^o for forecast evaluation
- For calculating EL, we need to choose θ: three different estimates
  - $\hat{\theta}^{(20)}_{\text{last month}}, \hat{\theta}^{(80)}_{\text{last month}}$: last month's empirical 20%/80%-quantile
  - $\min\{\hat{\theta}_{12 \text{ months}}\}$: minimum of last years's monthly 20%-quantiles

Transaction costs

Transaction cost-adjusted returns are calculated as follows:

$$to_t = \sum_{n=1}^{N} \left| w_{t+1}^{(n)} - w_t^{(n)} \frac{1 + r_t^{(n)}/100}{1 + w_t' r_t/100} \right|$$

Hence, total transaction costs on each dollar investment in month t are $c \cdot to_t$. The actual returns are then

$$r_{pt} = w_t' r_t - 100 \cdot c \cdot to_t$$

with c ranging from 0 to 15bps

In our setup

$$w_t^{(n)} = 0 \qquad \text{or} \qquad w_t^{(n)} \approx \frac{1}{130}$$

Portfolio performance

	ARV^o	Return	SR	TE	OO	SO	TO
Oracle RV^o	**36.75**	12.28	1.26	–	–	0.65	0.71
Avg. forecast	**45.09**	8.83	0.84	0.71	0.67	0.84	0.34
EL with $\hat{\theta}^{(20)}_{\text{last month}}$	**45.79**	9.05	0.87	0.66	0.67	0.79	0.44
EL with $\hat{\theta}^{(80)}_{\text{last month}}$	**46.03**	8.69	0.83	0.71	0.66	0.78	0.47
EL with $\min\{\hat{\theta}^{(20)}_{\text{last 12 months}}\}$	**46.06**	8.77	0.84	0.72	0.66	0.82	0.38
HAR (VIX)	**46.48**	9.05	0.87	0.70	0.64	0.83	0.36
QLIKE	**46.58**	8.64	0.81	0.68	0.67	0.78	0.47
HAR (RV^o_{66}, RV^o_{132})	**46.77**	9.16	0.87	0.72	0.65	0.81	0.41
EL with $\theta^{(80)}_{\text{rolling}}$	**46.81**	8.67	0.80	0.75	0.66	0.80	0.42
SE	**47.07**	8.59	0.81	0.66	0.66	0.78	0.46
AR (rank)	**48.37**	8.19	0.78	0.75	0.66	0.89	0.26
Last-month RV^o	**48.51**	7.75	0.74	0.76	0.65	0.65	0.71
Riskmetrics RV^d	**48.51**	8.03	0.78	0.72	0.65	0.94	0.15
Oracle RV^d	**49.26**	10.68	1.21	0.51	0.70	0.53	0.95
Last-month RV^d (benchmark)	52.46	8.54	0.82	0.77	0.59	0.53	0.95
Idio vola	**59.12**	8.55	0.70	1.26	0.50	0.47	1.08

Sharpe ratio (SR), tracking error (TE), oracle overlap (OO), self-overlap (SO), turnover (TO).
Blue: "best" model

Table: Excess returns for different values of c

	c = 5bps			c = 10bps			c = 15bps		
	TC	low	low-high	TC	low	low-high	TC	low	low-high
Oracle RV^o	0.43	**11.84**	**19.18**	0.86	**11.41**	**19.22**	1.30	**10.98**	**19.27**
Oracle RV^d	0.57	**10.11**	**16.71**	1.14	**9.55**	**16.72**	1.70	**8.98**	**16.72**
HAR (RV^o_{66}, RV^o_{132})	0.24	8.91	**10.90**	0.49	**8.67**	**10.91**	0.73	**8.43**	**10.92**
HAR (VIX)	0.22	8.83	10.47	0.43	8.62	10.49	0.65	**8.40**	10.49
EL with $\hat{\theta}^{(20)}_{\text{last month}}$	0.27	8.79	9.89	0.53	8.52	9.86	0.80	**8.26**	9.84
Avg. forecast	0.21	8.62	10.01	0.42	8.42	10.03	0.62	**8.21**	10.04
EL with $\min\{\hat{\theta}^{(20)}_{\text{last 12 months}}\}$	0.23	8.54	9.83	0.46	8.31	9.82	0.69	**8.08**	9.82
EL with $\hat{\theta}^{(80)}_{\text{last month}}$	0.28	8.41	10.00	0.57	8.12	9.98	0.85	7.84	9.97
QLIKE	0.28	8.36	9.87	0.56	8.08	9.86	0.84	7.80	9.86
SE	0.27	8.32	9.35	0.55	8.04	9.34	0.82	7.77	9.33
AR (rank)	0.15	8.04	8.62	0.31	7.88	8.66	0.46	7.73	8.71
Last-month RV^d (benchmark)	0.57	7.97	8.19	1.14	7.41	8.20	1.70	6.84	8.20
Idio vola	0.65	7.91	7.71	1.30	7.26	7.70	1.94	6.61	7.69
Last-month RV^o	0.43	7.32	7.44	0.87	6.89	7.49	1.30	6.46	7.53

Bold: Returns significantly different from benchmark portfolio Last-month RV^d, sorted by 5bps low-vola returns. Blue: "best" model

FFC and FF five-factor

	CAPM		FFC					FF five-factor					
	α	β_{MKT}	α	β_{MKT}	β_{SMB}	β_{HML}	β_{MOM}	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}
Oracle RV^o	7.579	0.622	7.352	0.680	-0.149	0.002	0.057	6.821	0.692	-0.129	-0.075	0.131	0.158
	[0.000]	[0.000]	[0.000]	[0.000]	[0.003]	[0.966]	[0.005]	[0.000]	[0.000]	[0.006]	[0.157]	[0.010]	[0.159]
Oracle RV^d	6.429	0.563	6.195	0.620	-0.134	0.009	0.068	5.768	0.628	-0.118	-0.081	0.112	0.181
	[0.000]	[0.000]	[0.000]	[0.000]	[0.006]	[0.798]	[0.001]	[0.000]	[0.000]	[0.016]	[0.096]	[0.026]	[0.032]
Avg. forecast	3.865	0.658	3.611	0.718	-0.138	0.000	0.071	3.044	0.731	-0.117	-0.099	0.142	0.213
	[0.003]	[0.000]	[0.001]	[0.000]	[0.005]	[0.999]	[0.003]	[0.018]	[0.000]	[0.029]	[0.114]	[0.018]	[0.096]
HAR (RV^o_{66}, RV^o_{132})	4.111	0.668	3.900	0.720	-0.121	0.023	0.070	3.344	0.733	-0.101	-0.078	0.140	0.220
	[0.002]	[0.000]	[0.003]	[0.000]	[0.012]	[0.612]	[0.001]	[0.009]	[0.000]	[0.048]	[0.217]	[0.013]	[0.071]
Last-month RV^d (benchmark)	3.404	0.680	3.200	0.729	-0.104	0.031	0.076	2.768	0.735	-0.087	-0.065	0.116	0.188
	[0.004]	[0.000]	[0.003]	[0.000]	[0.016]	[0.362]	[0.001]	[0.012]	[0.000]	[0.065]	[0.257]	[0.063]	[0.120]

Discussion

Results so far:

- HAR models lead to significant decrease of volatility inside low-volatility portfolios
- Avg. forecast strikes a good balance without selecting a particular model
- Selection based on loss functions is not necessarily beneficial compared to the avg. forecast
- Oracle portfolios' returns differ across measures of quadratic variation

Coming soon:

- Utility analysis
- Forecast-weighted portfolios

Thank you very much!

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