

# The Effects of Financing Rules in Pay-As-You-Go Pension Systems on the Life and the Business Cycle

Christian Scharrer<sup>a</sup>

<sup>a</sup>University of Augsburg, Department of Economics, Universitätsstrasse 16, 86159 Augsburg, Germany. E-mail: christian.scharrer@wiwi.uni-augsburg.de

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## **Abstract:**

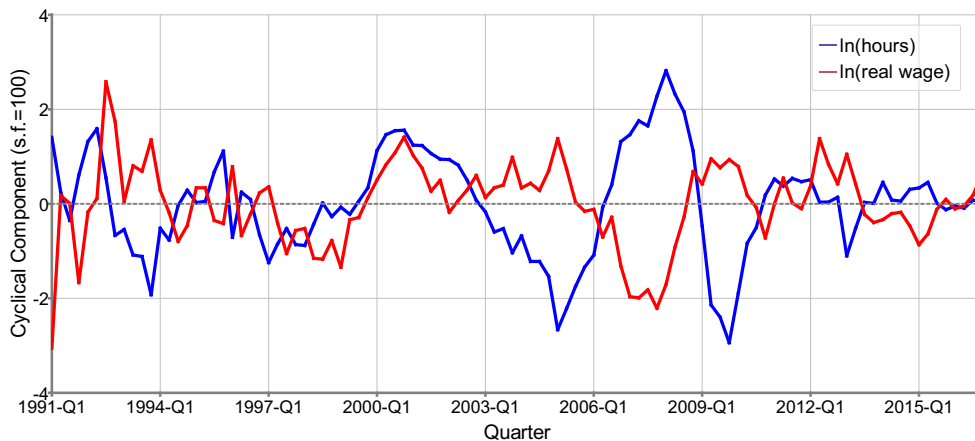
I study the impacts of financing rules for financial surpluses in pay-as-you-go pension systems on the business cycle and the life cycle in a dynamic stochastic large-scale overlapping generations model, where households take the inter-temporal links between contributions and pension benefits explicitly into account. The results point out that sluggish adjustments of contribution rates that are implemented by adjusting a financial buffer stock both stabilize an economy and decrease the volatility of life-time utilities of retirees and workers close to retirement. Such a policy allows these households a better hedge against macroeconomic shocks over the business cycle. Moreover, I show that the impacts of higher fluctuations of aggregate variables on the volatility of individual lifetime utilities can rather be negligible.

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# 1 Introduction

Revenues and pension benefits in pay-as-you-go (PAYG) pension schemes depend mainly on labor earnings which fluctuate over the business cycle. Fig. 1 shows the associated development of the cyclical component of aggregate hours and real wages for Germany from 1991:1 to 2016:4. Both variables vary substantially and, as a consequence, contribution rates, pension benefits and/or the stock of financial assets in a PAYG system have to be adjusted so that its budget is balanced. However, these adjustments affect the intergenerational allocation of income and aggregate risk which in turn influences the consumption smoothing behavior of households over the life cycle. For example, the social security authority could only adjust the contribution rates. Such a financing rule shifts macroeconomic risks to younger generations since it increases the volatility of net wages of workers and holds pension benefits constant. Workers are, however, better able to deal with higher economic risks by changing their labor supply and savings rates in response to macroeconomic shocks, whereas retirees can only adjust their savings rate.



**Figure 1:** The Cyclical Component of Real Wages and Hours (hp-filtered with weight 1600, s.f. = scaling factor, source: Destatis, own calculations).

In this paper, I study the effects of different financing rules for potential financial surpluses in PAYG systems on the business cycle and the age-specific consumption smoothing behavior of households in a large-scale real business cycle model with overlapping generations, where households take the inter-temporal link be-

tween contributions and pension benefits explicitly into account. In particular, I find that sluggish adjustments of contribution rates that are implemented by adjusting a buffer stock of financial assets of a PAYG system both stabilize an economy and help to decrease the volatility of (remaining) life-time utilities of retirees and workers close to retirement, in contrast to solely complete adjustments of contribution rates. Such a policy reduces the distortionary effects of labor taxation and also allows a more flexible accumulation of wealth over the life cycle, which helps future retirees to hedge better against macroeconomic shocks over the business cycle.

The most closely related papers to mine are Thøgersen (1998) and Wagener (2003). Thøgersen (1998) studies the effects of PAYG pensions programs on the intergenerational allocation of risk and welfare. He finds that defined contribution rates imply a lower income risk and higher ex-ante welfare across generations. In contrast, Wagener (2003) shows that different PAYG schemes are not comparable in a ex ante perspective due to different information sets and decisions over the life cycle. From an ex post perspective, he concludes that fixed replacement rates are preferable to defined contributions. They improve intergenerational risk-sharing and induce higher utility levels. Both studies, however, assume that labor supply is completely inelastic in their two-period overlapping generations models, where they exclude general equilibrium effects since all prices (respectively, their probability distributions) are fully exogenous. This paper extends the aforementioned research and presents a large-scale dynamic stochastic overlapping generations model that methodically builds on Ríos-Rull (1996) and Heer and Maußner (2012). In particular, the model features 240 generations, takes general equilibrium effects into account, and labor supply is endogenous so that workers can adjust their labor supply in response to changes in factor prices. This approach, therefore, allows to study in more detail the age-specific life cycle effects of different financing rules in PAYG systems which keep the social security budget balanced over the business cycle.

The rest of this paper is organized as follows. Section 2 describes and explains the model, which I calibrate in Section 3. The resulting steady state is discussed in Section 4, while Section 5 studies the effects of different financing rules in pay-as-you-go systems on aggregate variables and the consumption smoothing behavior of

households over the business cycle. Section 6 concludes.

## 2 The Model

In this section, I present a model with overlapping generations and aggregate uncertainty, where the period length is set to one quarter. Households optimize their expected life-time utility, firms maximize profits, and a PAYG system transfers resources across generations.

### 2.1 Demographics

Each year, a new cohort is born and its size  $\psi_s$  is constant at age  $s = 1$  (corresponding to a real life age of 21). Households live at most  $T$  quarters, where they work in the first  $T_w$  quarters and are retirees in the subsequent  $T_r = T - T_w$  periods. In addition, each  $s$ -year old household survives from age  $s$  to  $s + 1$  with an exogenously given probability of  $\phi_s$ , where  $\phi_0 \equiv 1$ . Thus, the mass of households  $\psi_{s+1}$  at age  $s + 1$  evolves according to  $\psi_{s+1} = \phi_s \psi_s$ . For simplification, I normalize the total mass of living households  $\sum_{s=1}^T \psi_s$  to one.

### 2.2 Households

A household at age  $s = 1$  in period  $t$  maximizes the following discounted expected lifetime utility  $U_t$  with respect to consumption  $c_t^s$  and labor supply  $n_t^s$ :

$$U_t = E_t \sum_{s=1}^T \beta^{s-1} \left( \prod_{j=1}^s \phi_{j-1} \right) u(c_{t+s-1}^s, n_{t+s-1}^s), \quad (1)$$

where  $n_t^s \in [0, 1]$  for  $s \leq T_w$  and  $n_t^s \equiv 0$  for  $s > T_w$ . Moreover, the specification of the instantaneous utility function  $u(c_t^s, n_t^s)$  follows Trabandt and Uhlig (2011),

$$u(c_t^s, n_t^s) = \begin{cases} \ln(c_t^s) - \frac{\gamma_0}{1+1/\gamma_1} (n_t^s)^{1+1/\gamma_1}, & \text{for } \eta = 1, \\ \frac{1}{1-\eta} \left[ (c_t^s)^{1-\eta} \left( 1 - \frac{\gamma_0(1-\eta)}{1+1/\gamma_1} (n_t^s)^{1+1/\gamma_1} \right)^\eta - 1 \right], & \text{for } \eta \neq 1. \end{cases} \quad (2)$$

These preferences feature a constant Frisch elasticity of labor supply  $\gamma_1$  and a constant intertemporal elasticity of substitution  $1/\eta$ . The parameter  $\gamma_0$  controls the labor supply in the steady state of the model.

Households at age  $s = 1$  are born without assets and accumulate a stock of capital  $k_{t,j}^s$  over their life cycle. Their capital earns the real interest rate  $r_t$  and depreciates at the rate  $\delta$ . Moreover, I assume that the oldest households at age  $s = T$  leave no bequests and are not allowed to die indebted. The net labor income of workers depends on the real wage  $w_t$ , the age-specific productivity  $e^s$ , and the contribution rate  $\tau_t^p$  for the PAYG system, where pensions  $pens_t^s$  are only paid to retired agents. The government collects all accidental bequests and transfers them lump-sum back to the households in the form of  $tr_t$ . The respective budget constraint of a  $s$ -year old household in period  $t$  is given by

$$c_t^s + k_{t+1}^{s+1} = \begin{cases} (1 + r_t - \delta) k_t^s + (1 - \tau_t^p) w_t e^s n_t^s + tr_t, & \text{for } s \leq T_w, \\ (1 + r_t - \delta) k_t^s + pens_t^s + tr_t, & \text{for } s > T_w, \end{cases} \quad (3)$$

with  $k_t^1 = k_t^{T+1} \equiv 0$ ,  $n_t^s \equiv 0$  for  $s > T_w$ , and  $pens_t^s \equiv 0$  for  $s \leq T_w$ .

Pension entitlements  $pent_t^s$  depend on average lifetime labor earnings and an exogenously given replacement ratio  $\zeta$ . For ease of notation, I also introduce the parameter  $\theta_t$  which is equal to one in the steady state and can be adjusted by the social security authority such that it controls the effective replacement ratio  $\zeta\theta_t$  in period  $t$  outside the steady state. Thus, pension benefits are represented by

$$pens_t^s = \theta_t pent_t^s, \quad (4)$$

where pension entitlements can be expressed as

$$pent_t^s = \begin{cases} \frac{\zeta}{T_w} \sum_{i=1}^{T_w} w_{t-i} e^{T_w-i+1} n_{t-i}^{T_w-i+1}, & \text{for } s = T_w + 1, \\ pent_{t-s+T_w}^{T_w+1}, & \text{for } s > T_w + 1. \end{cases} \quad (5)$$

The representative first-order conditions that solve the optimization problems of

households consist of the aforementioned budget constraints (3) and

$$\lambda_t^s = \frac{\partial u(c_t^s, n_t^s)}{\partial c_t^s}, \quad (6)$$

$$\lambda_t^s = \beta \phi_s E_t \{ \lambda_{t+1}^{s+1} (1 + r_{t+1} - \delta) \}, \quad (7)$$

$$0 = \frac{\partial u(c_t^s, n_t^s)}{\partial n_t^s} + (1 - \tau_t^p) w_t e^s n_t^s \lambda_t^s + E_t \left\{ \sum_{a=T_w+1}^T \beta^{a-s} \left( \prod_{j=s+1}^a \phi_{j-1} \right) \lambda_{t+a-s}^a \frac{\zeta \theta_{t+a-s} w_t e^s}{T_w} \right\}. \quad (8)$$

The variable  $\lambda_t^s$  denotes the Lagrange multiplier.

## 2.3 Production

Aggregate Output  $Y_t$  is characterized by a Cobb-Douglas production function,

$$Y_t = Z_t N_t^{1-\alpha} K_t^\alpha. \quad (9)$$

The variables  $N_t$  and  $K_t$  denote aggregate labor and capital, respectively. Moreover, the stochastic technology level  $Z_t$  follows a standard AR(1) process:  $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$ , where  $\epsilon_t \sim N(0, \sigma^2)$ . The corresponding profit maximization under perfect competition implies zero profits and that factor rewards equal their marginal products,

$$w_t = (1 - \alpha) Z_t \left( \frac{K_t}{N_t} \right)^\alpha, \quad (10)$$

$$r_t = \alpha Z_t \left( \frac{K_t}{N_t} \right)^{\alpha-1}. \quad (11)$$

## 2.4 Social Security & Government

The government, in form of a social security authority, collects contributions at the rate  $\tau_t^p$  of gross labor incomes of workers and holds a buffer stock of financial assets

$F_t$  which invests in the capital market. Moreover, the age-specific public pension entitlements  $pent_t^s$  adjust over time since they depend on gross pre-retirement earnings and the steady state replacement ratio  $\zeta$  according to equation (5). These entitlements give in turn the pension benefits  $pens_t^s = \theta_t pent_t^s$  that can be adjusted in the short run by the variable  $\theta_t$ , as also described in equation (4). Thus, the budget of the PAYG system is given by

$$Pens_t + F_{t+1} = \tau_t^p w_t N_t + (1 + r_t - \delta) F_t, \quad (12)$$

where

$$Pens_t = \theta_t Pent_t, \quad (13)$$

$$Pent_t = \sum_{s=T_w+1}^T \psi_s pent_t^s, \quad (14)$$

$$N_t = \sum_{s=1}^{T_w} \psi_s e^s n_t^s. \quad (15)$$

In order to describe the dynamics of the variables  $\tau_t^p$ ,  $F_{t+1}$ , and  $\theta_t$  around the steady state, I follow a similar approach as in Galí et al. (2007) for fiscal policy rules and specify the following financing rule in the PAYG scheme,

$$F_{t+1} - F = \omega_F S_t, \quad (16)$$

$$\theta_t Pent_t - Pent_t = \omega_R (1 - \omega_F) S_t, \quad (17)$$

with

$$S_t = \tau^p (w_t N_t - wN) + (R_t F_t - RF) - (Pent_t - Pent) \quad (18)$$

and  $\omega_F, \omega_R \in [0, 1]$ . The expression  $R_t \equiv 1 + r_t - \delta$  in equation (18) defines the gross interest rate. Moreover, variables without a time index denote the corresponding steady state values. The term  $S_t$  describes potential surpluses in the PAYG system under the assumption that the PAYG administration keeps the effective replacement ratio and the contribution rate constant,  $\zeta \theta_t = \zeta \theta$  and  $\tau_t^p = \tau^p$ . Hence, the exogenous

parameters  $\omega_F$  and  $\omega_R$  in equation (16) and (17) control the adjustments of financial assets and effective replacement ratios over the business cycle. If, for example, both parameters are equal to zero, the budget constraint (12) implies that only the contribution rate  $\tau_t^p$  is allowed to change in order to keep the budget balanced. In contrast, a high value for  $\omega_F$  results in pronounced adjustments of financial assets which dampen the associated budget effects on the other variables.

Furthermore, I assume that all accidental bequests are collected by the government and transferred as lump-sums to the household sector. This implies

$$tr_t = \sum_{s=1}^T (1 - \Phi_{s-1}) \psi_{s-1} [(1 + r_t - \delta) k_t^s]. \quad (19)$$

## 2.5 Equilibrium

In a general equilibrium, individual and aggregate behavior must be consistent. Thus, the following conditions have to be satisfied for all  $t$ ,

$$N_t = \sum_{s=1}^{T_w} \psi_s n_t^s, \quad (20a)$$

$$K_t = \sum_{s=1}^T \psi_{s-1} k_t^s + F_t, \quad (20b)$$

$$C_t = \sum_{s=1}^T \psi_s c_t^s, \quad (20c)$$

such that the goods market clears:

$$Z_t N_t^{1-\alpha} K_t^\alpha = C_t + I_t, \quad (21)$$

where  $I_t = K_{t+1} - (1 - \delta)K_t$ .



### 3 Calibration

I calibrate the model on a quarterly basis for the German economy and linearize the model around the steady state.<sup>1</sup> Households live at most  $T = 240$  quarters and work for  $T_w = 160$  quarters such that they enter retirement at a real life age of 61 years and die with certainty at a real life age of 81 years. These numbers roughly correspond with the average age when households enter retirement and the average life expectancy for men and women for the year 2011 in Germany, see Deutsche Rentenversicherung Bund (2017). Moreover, I approximate the average survival probabilities  $\phi_s$  with German life tables for the sample 1992 to 2012 which are provided by the Federal Statistical Office (Destatis). The smoothed productivity profiles  $e^s$  of  $s$ -year-old workers are taken from Heer (2019), who calculates these profiles with data of age-specific hourly wages during 1990 and 1997 for Germany.

With respect to the production technology, I use values estimated by Flor (2014) for the German economy for the sample 1991:1 to 2012:4. The production elasticity of capital is equal to  $\alpha = 0.34$  and the depreciation rate  $\delta$  equals 1.7 percent. Moreover, the autocorrelation parameter for technology shocks is set to  $\rho = 0.83$  and the corresponding standard deviation of innovations is equal to  $\sigma = 0.0082$ , where Flor (2014) takes both capital and labor as factor inputs into account for the calculation of the Solow residual.

The parameters describing the PAYG system are chosen as follows: The replacement ratio  $\zeta$  of pensions relative to average pre-retirement earnings is set to 42 percent and taken from DICE Database (2016) for the year 2011. Moreover, I assume that the stock of financial assets  $F$  is equal to aggregate (quarterly) pensions entitlements  $Pent$  in the steady state and set the parameter  $\theta$ , which controls the effective replacement ratio  $\zeta\theta_t$  in period  $t$ , equal to one, respectively. The resulting stationary contribution rate amounts to  $\tau^p = 16$  percent, which is a little bit lower than

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<sup>1</sup>In particular, I use the solution methods described in Chapters 9 and 10 in Heer and Maußner (2009) and modified codes of the provided CoRRAM package (see [www.wiwi.uni-augsburg.de/vwl/maussner/dge\\_buch/dge\\_book\\_2ed/downloads\\_2nd](http://www.wiwi.uni-augsburg.de/vwl/maussner/dge_buch/dge_book_2ed/downloads_2nd)).

its empirical counterpart of 19.90 percent for the year 2011.<sup>2</sup> With respect to the parameters  $\omega_F$  and  $\omega_R$ , I distinguish between three cases that I will discuss in the following sections:

- **Case 1:** This is the benchmark case, where I assume that the PAYG authority seeks to keep the contribution rate  $\tau_t^p$  as constant as possible and does not adjust the effective replacement ratio  $\zeta\theta_t$  over the business cycle.<sup>3</sup> For that reason, I set the parameter  $\omega_P = 0$  and  $\omega_F = 0.95$ .<sup>4</sup>
- **Case 2:** In this case, the PAYG authority chooses  $\omega_P = 1$  and  $\omega_F = 0$  so that only the effective replacement ratio  $\zeta\theta_t$  fluctuates over the business cycle.
- **Case 3:** Here, the PAYG system only adjusts the contribution rates,  $\omega_F = \omega_R = 0$ , where the stock of financial assets and the effective replacement ratio stay constant.

Regarding the preference parameters, I set the discount factor  $\beta$  equal to 1.00 such that the real rate of return on capital,  $r_t - \delta$ , equals a value of 4 percent which describes the long term average according to Busl and Seymen (2013). Furthermore, the parameter  $\gamma_0 = 4.34$  implies an average labor supply in the steady state of 0.33. With respect to the Frisch labor supply elasticity, I choose  $\gamma_1 = 2.15$  in order to roughly match a relative volatility of aggregate hours to aggregate output of 0.62 for the benchmark case according to Flor (2014). This value is in line with the macro-economic literature, which often uses Frisch elasticities between 2 and 4.<sup>5</sup> Furthermore, I choose a standard value of 2 for the parameter  $\eta$  implying an intertemporal elasticity of substitution of 0.5.

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<sup>2</sup>See Deutsche Rentenversicherung Bund (2017).

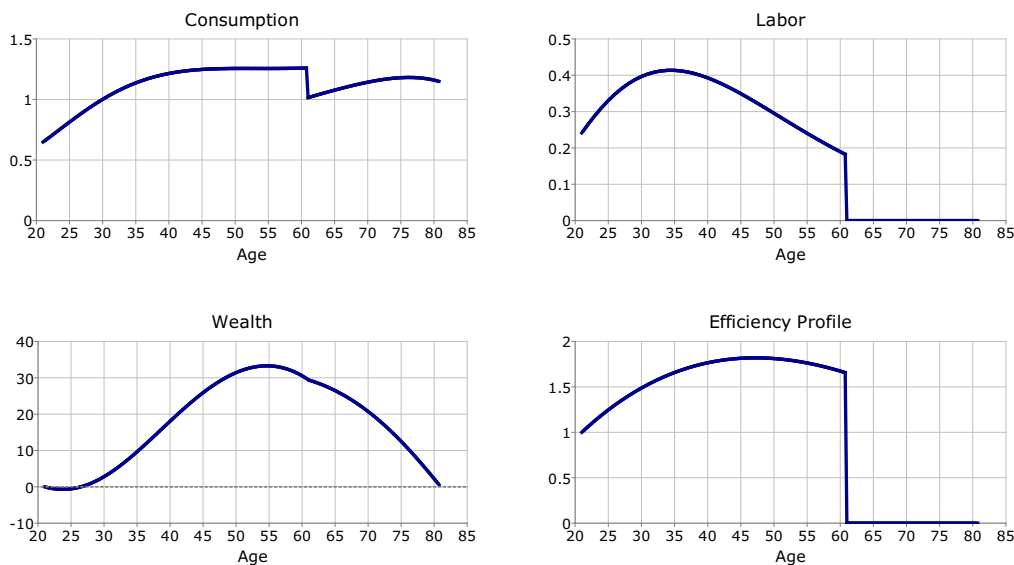
<sup>3</sup>In Germany, the PAYG administration adjusts the contribution rates when the size of the reserve fund (Nachhaltigkeitsrücklage) exceeds (undershoots) the monthly expenditures by 150 (20) percent, see §158 SGB VI.

<sup>4</sup>This calibration still ensures local stability around the steady state and allows to approximate this form of financing. Then, the PAYG administration mainly changes its stock of financial assets and dampens the adjustments of contribution rates.

<sup>5</sup>See, for example, Peterman (2016).

## 4 Steady State

Fig. 2 presents the behavior of households over the life-cycle in the steady state. The consumption profile in the upper left panel increases until an age of 60 years and displays a kink when households enter retirement due to the increase of leisure that in turn increases the marginal utility of consumption.<sup>6</sup> Their labor supply, as displayed in the upper right panel, increases during the first 15 years and falls monotonously thereafter when income and wealth effects start to dominate the substitution effects. Moreover, the lower left panel shows that households build up wealth for retirement until an age of about 55 years and start to decrease their stock of capital in the following periods in order to smooth their consumption over the life cycle. The age-specific efficiency profile also follows a hump-shaped pattern and is displayed in the lower right panel.



**Figure 2:** Steady-State Behavior of Households (abscissa: age in years).

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<sup>6</sup>For the reader's convenience, I use the real life age in years in contrast to the quarterly age index  $s$  in the discussions and figures hereinafter.

## 5 Effects over the Business Cycle

In this section, I study how financing rules in PAYG systems affect aggregate variables and the consumption smoothing behavior of households. Fig. 3 presents the associated impulse responses of aggregate variables to a positive one-time productivity shock of one standard deviation in period  $t = 2$ .

The first two rows in Fig. 3 show the benchmark case. A technology shock increases output, labor supply, consumption, and investment. The real interest rate rises due to the increase in productivity and labor supply. Moreover, both the rise of average productivity and the increase of the stock of capital in the subsequent periods dominate the negative effects of labor supply increases on the marginal product of labor so that the real wage rate also rises, as illustrated in the upper right panel of Fig. 3. Furthermore, the increase in labor incomes leads to financial surpluses in the PAYG system and growing pension entitlements. For Case 1, the panels of the second row show, in particular, that the PAYG administration mainly invests these surpluses in financial assets in order to dampen the reduction in contribution rates.

The impulse responses for Case 2 and 3 are plotted in the last four rows of Fig. 3. Overall, the behavior of aggregate variables in Case 2 is almost identical in comparison with Case 1, while the amplitudes of impulse responses in Case 3 are a little bit more pronounced. For example, output  $Y_t$  increases on impact by 1.38 percent in Case 1, 1.39 percent in Case 2, and by 1.60 percent with respect to Case 3, whereas the contribution rates decline by 0.07, 0, and 1.64 percent in Cases 1 to 3, respectively. The economic intuition for these results is straightforward. On the one hand, pronounced adjustments of contribution rates in Case 3 induce stronger distortionary effects on individual labor supply decisions and, therefore, result in larger fluctuations of aggregate labor supply and real output. On the other hand, the share of financial assets only amounts to 0.84 percent of aggregate capital in the steady state. For that reason, the associated distortionary impacts of asset changes in a PAYG system in Case 1 on real factor prices are rather negligible so that the results in Case 1 and 2 are very similar. Thus, if the PAYG authority solely adjusts the contribution rates, it increases these distortionary effects, while the other financ-

ing forms help to stabilize the economy by keeping the contribution rates (almost) constant.

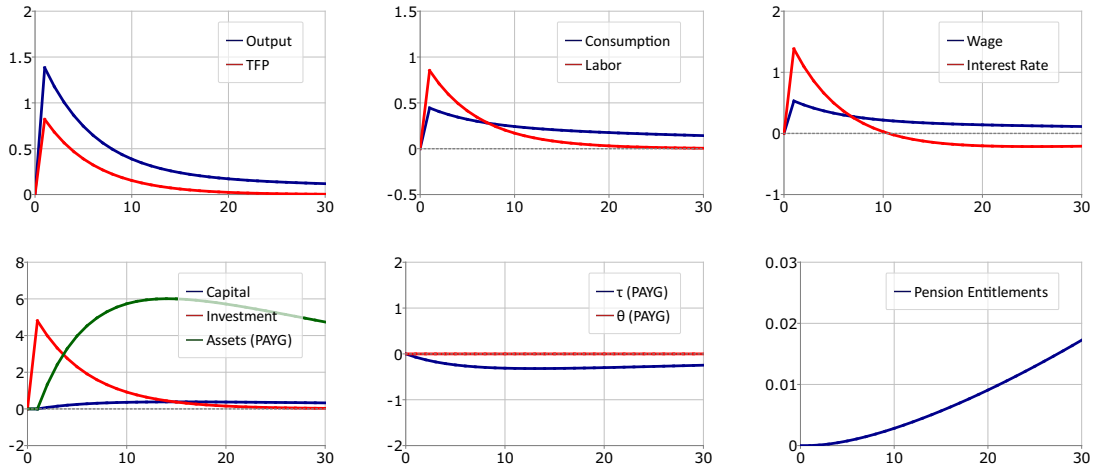
Table 1, which displays the simulated standard deviations of aggregate variables and their empirical counterparts for the sample 1991:1 to 2012:4, also confirms the previous results. Financing rules, which try to keep the contributions rates mostly constant, imply lower volatilities of aggregate output, labor, and consumption. For example, the second column shows that the standard deviation of aggregate output amounts to 1.64 in Case 1 and increases by 16 percent to 1.91 in Case 3, whereas it almost stays constant with regard to Case 2. Comparing our benchmark model in the first two rows with empirical data in the last two rows in Table 1, we can, moreover, see that the benchmark model produces standard characteristics of business cycle volatilities which roughly match the data regarding output and labor. However, the standard deviations of investment, consumption, and the real wage are a little bit more different than their empirical values.

	$Y$	$N$	$I$	$C$	$w$	$\tau^P$
Case 1:	1.64	1.02	5.79	0.53	0.63	0.25
	(1.00)	(0.62)	(3.52)	(0.32)	(0.38)	(0.15)
Case 2:	1.66	1.04	5.71	0.58	0.62	0.00
Case 3:	1.91	1.42	6.45	0.69	0.50	1.97
Data*:	1.51	0.93	4.35	0.77	0.79	.
	(1.00)	(0.62)	(2.88)	(0.51)	(0.52)	.

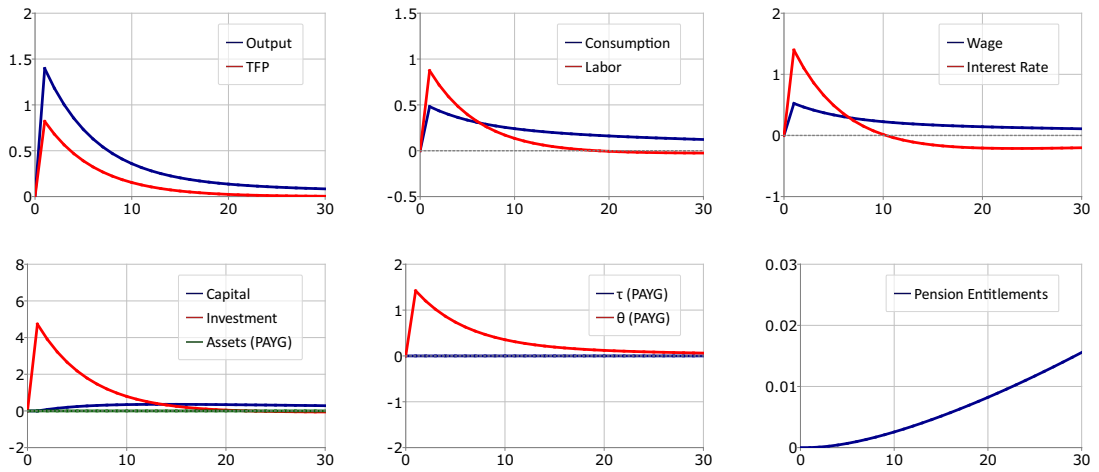
**Table 1:** Standard Deviations of Aggregate Variables (time series were hp-filtered using a parameter of 1600 over 100,000 simulations with a period length of 88 quarters; in parentheses: relative deviations with respect to output; \*sample: 1991:1-2012:4, source: Flor (2014)).

The previous findings clearly suggest that financing forms aiming to keep the contribution rates of a PAYG scheme (nearly) constant stabilize an economy. These financing rules, however, have different effects on the intergenerational allocation of risk and affect the consumption smoothing behavior of households over the life cycle. For this reason, I simulate 1,000,000 periods with the same sequence of random total factor productivity shocks for Case 1 to 3 and compute the standard deviations

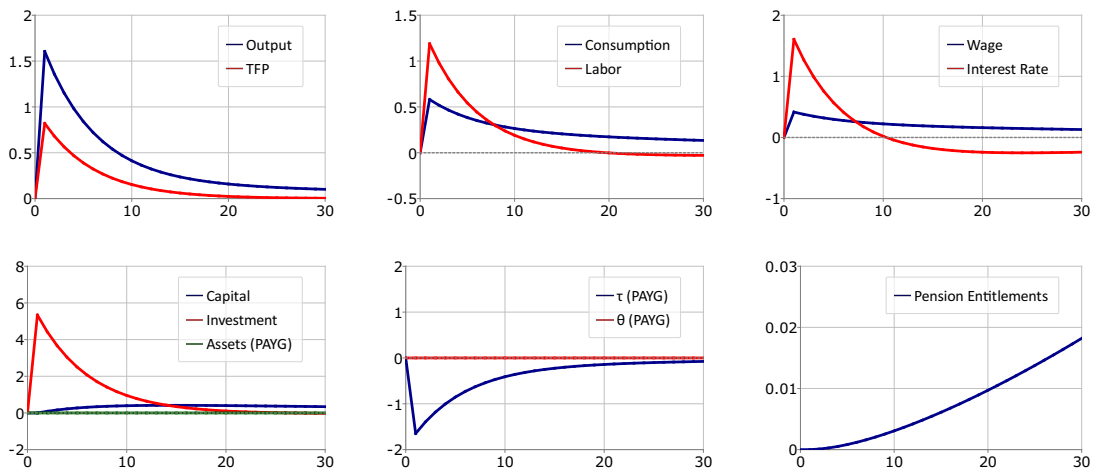
Case 1 - Adjustment of Assets and Contribution Rates ( $\omega_F = 95\%$  and  $\omega_R = 0\%$ ):



Case 2 - Adjustment of Replacement Ratios ( $\omega_F = 0\%$  and  $\omega_R = 100\%$ ):



Case 3 - Adjustment of Contribution Rates ( $\omega_F = 0\%$  and  $\omega_R = 0\%$ ):



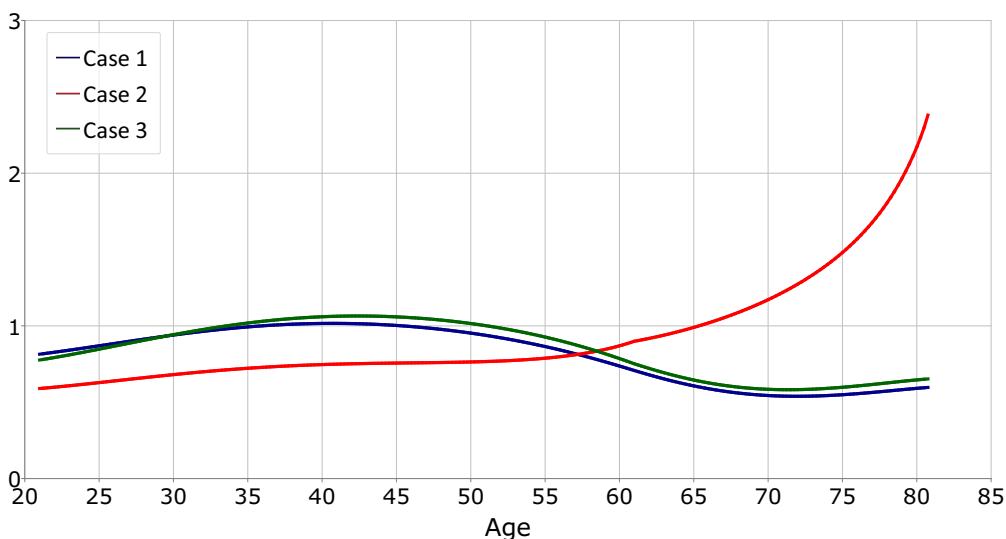
**Figure 3:** Impulse Responses of Aggregate Variables (ordinate: percent deviations, abscissa: periods).

of (remaining) ex-post lifetime utilities of all households at ages 21 to 80, which I express as consumption equivalent changes (CECs).<sup>7</sup>

By comparing the volatilities of consumption equivalent changes in Fig. 4 for each case, we can see that sole adjustments of replacement ratios in Case 2 are associated with very low volatilities in lifetime utility for workers and very high volatilities in life time utility of retirees since this financing form shifts the macroeconomic risk towards retirees. In contrast, the social security authority only adjusts the contributions rates in Case 3 and, therefore, burdens current working households with larger fluctuations in their net income, whereas the pronounced adjustments of the stock of financial assets in Case 1 dampen these income fluctuations in Case 3. However, it is interesting that the standard deviations of lifetime utilities almost follow the same patterns in Case 1 and 3, where the volatility of consumption equivalent changes is slightly lower for households who are older than 30 years in Case 1. Thus, households are able to deal with both financing forms almost equally well by accordingly changing their labor supply and savings decisions over the life cycle, even though Case 3 causes more pronounced fluctuations of aggregate variables, as shown in Table 1. The effects of larger fluctuations of aggregate variables on the consumption smoothing behavior of households, therefore, seem to be negligible. Furthermore, in comparison to Case 2, the volatilities in Cases 1 and 3 are slightly higher up to an age of about 57 years and considerably lower for older age groups. For example, the standard deviation of the youngest (oldest) household amounts to 0.81 (0.59) and 0.78 (0.65) percent in Case 1 and 3, while it is equal to a value of 0.58 (2.38) percent with respect to Case 2.

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<sup>7</sup>The consumption equivalent change describes the percentage variation of steady state consumption that is equivalent to a given change in intertemporal welfare.



**Figure 4:** Volatilities of Consumption Equivalent Changes (ordinate: s.d. of CECs in percent, abscissa: age in years).

## 6 Conclusion

The analysis in this paper has shown how financing rules for additional surpluses in a PAYG system affect aggregate variables and the consumption smoothing behavior of households over the business cycle. Financing forms that keep the contribution rates in a PAYG system (almost) constant imply in general lower absolute standard deviations of aggregate output, labor, investment, and consumption in comparison to complete adjustments of contribution rates. However, the effects on the volatility of (remaining) life-time utilities can be very different. On the one hand, sole adjustments of current replacement ratios, which mainly burden old generations due to constant contribution rates, result in very low fluctuations in lifetime utility of very young households and very high fluctuations for retirees. On the other hand, a financing rule that mainly adjusts the stock of financial assets of a PAYG scheme in order to avoid large fluctuations of contributions rates increases slightly the volatility of life-time utilities of young households, but also implies much lower volatilities for workers close to retirement and retirees. Moreover, complete adjustments of contribution rates yield only slightly higher standard deviations of lifetime utilities of households who are older than 30 years with respect to the aforementioned fi-



nancing rule, despite much larger fluctuations in aggregate variables. Consequently, the impacts of higher standard deviations of aggregate variables on the volatility of individual welfare can rather be negligible. One should, however, be careful to use these welfare results for normative conclusions since no financing form studied in this paper strictly dominates the other.

## **Compliance with ethical standards**

Conflict of Interest: The author declares that he has no conflict of interest.

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